Extraction of Plumes in Turbulent Thermal Convection

Emily S. C. Ching,[1] H. Guo,[1,2] Xiao-Dong Shang,[1,3] P. Tong,[3,4] and and Ke-Qing Xia[1]

[1]*Department of Physics, The Chinese University of Hong Kong, Shatin, Hong Kong, People's Republic of China*
[2]*State Key Laboratory for Studies of Turbulence and Complex Systems, Dept. of Mech. and Engin. Sci., Peking University, Beijing 100871, People's Republic of China*
[3]*Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078, USA*
[4]*Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, People's Republic of China*

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We present a scheme to extract the velocity of buoyant structures in turbulent thermal convection from simultaneous local velocity and temperature measurements. Applying this scheme to measurements taken at positions within the convection cell where the buoyant structures are dominated by plumes, we obtain the temperature dependence of the plume velocity and understand our results using the equations of motion. We further obtain the scaling behavior of the average local heat flux in the vertical direction at the cell center with the Rayleigh number and find that the scaling exponent is different from that measured for the Nusselt number. This difference leads to the conclusion that heat cannot be mainly transported through the central region of the convection cell.

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The Rayleigh-Bénard convection system consists of a closed cell of fluid heated from below and cooled on the top. The equations of motion, in Boussinesq approximation, are [1]:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \nu\nabla^2\mathbf{v} + g\alpha\delta T\hat{\mathbf{z}} \qquad (1)$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \kappa\nabla^2 T \qquad (2)$$

$$\nabla\cdot\mathbf{v} = 0, \qquad (3)$$

where $\mathbf{v}$ is the velocity field, p the pressure divided by density, T the temperature field, and $\hat{\mathbf{z}}$ is the unit vector in the vertical direction. Furthermore, $\delta T = T - T_0$, where T_0 is the mean temperature of the bulk fluid, g is the acceleration due to gravity and α, ν, and κ are, respectively, the volume expansion coefficient, kinematic viscosity, and thermal diffusivity of the fluid. The state of fluid motion is characterized by the geometry of the cell and two dimensionless parameters: the Rayleigh number, $\mathrm{Ra} = \alpha g\Delta L^3/(\nu\kappa)$, which measures how much the fluid is driven and the Prandtl number, $\mathrm{Pr}= \nu/\kappa$, which is the ratio of the diffusivities of momentum and heat of the fluid. Here, Δ is the maintained temperature difference between the bottom and the top, and L is the height of the cell. When Ra is sufficiently large, the convective motion becomes turbulent.

In turbulent convection, velocity and temperature measurements taken at a point within the convection cell display complex fluctuations in time. On the other hand, visualization of the flow reveals recurring coherent structures. One prominent coherent structure is a plume, which is a flow structure generated from the thermal boundary layers by buoyancy. Thus, at least two strategies can be employed to study turbulent thermal convection or turbulent flows in general. One is to analyze and understand the fluctuations of the local measurements. The other is to characterize the coherent structures and study and understand their dynamics. These two approaches are not independent but provide complementary knowledge of the flows. In particular, there is the natural question of whether and how information about the coherent structures can be extracted from the local measurements.

For turbulent flows not driven by buoyancy, various methods including proper orthogonal decomposition, conditional sampling, and wavelet analysis have been proposed to identify coherent vortical structures from local velocity measurements [2]. On the other hand, much less work has been done in identifying or extracting information about plumes in turbulent thermal convection [3–5]. Belmonte and Libchaber [3] used the skewness of the temperature derivative as a signature of the plumes. Zhou and Xia [4] associated the difference in the skewness of the positive and negative parts of the temperature difference with the presence of plumes and identified the plumes whenever the temperature difference is larger than a chosen threshold. In Ref. [5], plumes are identified when each of the temperature, vertical velocity, or vertical vorticity is larger than some threshold.

In this Letter, we present a scheme to extract the velocity of buoyant structures using simultaneous local velocity and temperature measurements. Our method makes explicit use of the physical intuition that these flow structures are generated by buoyancy and their velocity should thus be related to the temperature fluctuation in some way.

The experimental measurements studied were taken at the center and near the sidewall (on the midplane, at 8 mm from the sidewall) of an aspect-ratio-one cylindrical cell of height $L = 20.5$ cm and filled with water [6]. Velocity $\mathbf{v}(t)$ was measured using a two-component laser Doppler velocimetry (LDV) system [7] while temperature $T(t)$ was measured using a thermistor of 0.2 mm in diameter, 15 ms in time constant, and 20 mK/Ω in temperature sensitivity. A multichannel LDV interface module was used to synchronize the velocity and temperature data acquisition [6]. The spatial separation between the LDV focusing spot and the thermistor tip is 0.7 ± 0.2 mm, a distance smaller than the correlation length between the temperature and velocity fluctuations [8]. This allows us to assume that the simultaneous velocity and temperature measurements were taken at the same location. Near the sidewall, the vertical velocity component v_z and one horizontal velocity component v_y out of the rotation plane of the mean large-scale circulation were measured. At the cell center, all three velocity components v_x, v_y, and v_z were measured with the x-direction along that of the mean large-scale circulation near the bottom plate.

From the measurements, we calculate $\langle \mathbf{v}|T(t)\rangle$, the conditional average of the velocity on the temperature measured at time t. This is done as follows. At each time t, we find the times t' at which the temperature measurements $T(t')$ have values falling within $T(t) \pm 0.005$ °C. Then we calculate the average of those velocity measurements $\mathbf{v}(t')$ at such times t'. The resulting quantity is $\langle \mathbf{v}|T(t)\rangle$. Next, we decompose $\mathbf{v}(t)$ into a sum of $\langle \mathbf{v}|T(t)\rangle$ and the remaining part, denoted as $\mathbf{v_b}(t)$:

$$\mathbf{v}(t) = \langle \mathbf{v}|T(t)\rangle + \mathbf{v_b}(t). \tag{4}$$

Since $\langle \mathbf{v} f(T)\rangle = \langle\langle \mathbf{v}|T(t)\rangle f(T)\rangle$ for any function f of T [9], $\langle \mathbf{v_b}(t) f(T)\rangle = 0$. That is, $\mathbf{v_b}(t)$ is uncorrelated with any function of $T(t)$ and averages to zero over time. Hence, we interpret $\mathbf{v_b}(t)$ as the background velocity fluctuation. In the presence of buoyant structures such as plumes, the velocity field $\mathbf{v}(t)$ would correlate with *some* function of $T(t)$. As a result, the conditional average $\langle \mathbf{v}|T(t)\rangle$ will be different from the ordinary average velocity $\langle \mathbf{v}\rangle$, and is a function of T as well as t [since $T(t)$ depends on t]. In this case, we take the velocity of the buoyant structures to be $\langle \mathbf{v}|T(t)\rangle$. In other words, we associate the velocity of the buoyant structures with the part of $\mathbf{v}(t)$ that correlates with *some* function of $T(t)$. This is in accord with the physical intuition that the velocity of the buoyant structures must be related to the temperature fluctuation in some way. On the other hand, if $\mathbf{v}(t)$ is uncorrelated with *any* function of $T(t)$, then $\langle \mathbf{v}|T(t)\rangle = \langle \mathbf{v}\rangle$ and no buoyant structures are identified.

At the center and near the sidewall of the convection cell, $\langle \mathbf{v}|T(t)\rangle$ is indeed different from $\langle \mathbf{v}\rangle$. Flow visualization shows that the dominant buoyant structures at these locations are plumes. Therefore, we take the velocity of the plumes to be $\mathbf{v_p}(t) \equiv \langle \mathbf{v}|T(t)\rangle$. Moreover, the difference between $\langle \mathbf{v}|T(t)\rangle$ and $\langle \mathbf{v}\rangle$ lies mainly in the vertical direction. Thus, we have $\mathbf{v_p}(t) \approx v_{pz}(t)\hat{\mathbf{z}}$. We can also easily identify the local heat flux carried by the plumes. The normalized local heat flux is given by $\mathbf{j}(t) = [\mathbf{v}(t)\delta T(t)]L/(\kappa\Delta)$. The part carried by the plumes is thus $\mathbf{j_p}(t) \equiv [\mathbf{v_p}(t)\delta T(t)]L/(\kappa\Delta)$. When the contribution by the plumes is subtracted, the remaining is the heat flux carried by the background velocity fluctuation, which is $\mathbf{j_b}(t) = [\mathbf{v_b}(t)\delta T(t)]L/(\kappa\Delta)$ and $\langle \mathbf{j_b}(t)\rangle = 0$.

It was recently found [6] that the probability density function (pdf) of the local heat flux in the horizontal direction is approximately symmetric whereas that in the vertical direction is skewed towards the positive fluctuation. This asymmetry in the vertical flux pdf was interpreted [6] to be caused by the plumes. We use these results to test our method: we expect that after subtracting the contribution by the plumes, the pdf of the local heat flux in the horizontal and vertical directions would become identical. In Fig. 1, we show the pdf of $\tilde{j}_{bx}$, $\tilde{j}_{by}$, and $\tilde{j}_{bz}$. Here, the standardized variable $\tilde{X}$ is defined as

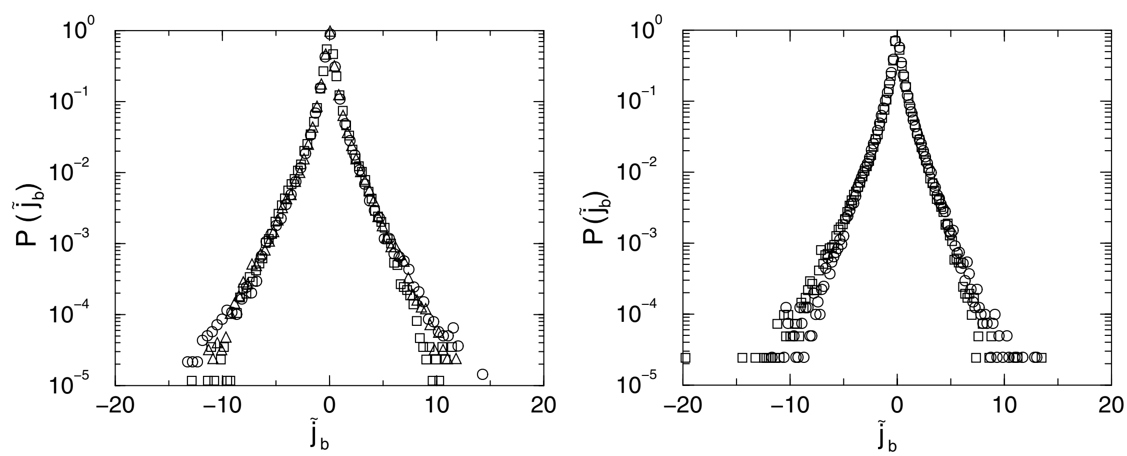

FIG. 1. The standardized pdf $P(\tilde{j}_{bx})$ (triangles), $P(\tilde{j}_{by})$ (squares), and $P(\tilde{j}_{bz})$ (circles) at $\text{Ra} = 2.6 \times 10^9$. The measurements were made at the cell center (left) and near the sidewall (right).

$(X - \langle X \rangle)/\sigma_X$ where $\langle X \rangle$ and σ_X are, respectively, the mean and standard deviation of X. In the calculation, we estimate T_0 by the average temperature at the cell center. As expected, the standardized pdfs of the horizontal and vertical components of $\tilde{\mathbf{j}}_\mathbf{b}$ are the same. Moreover, they are approximately the same at the two locations studied.

In another study [10], the velocity fluctuations at the cell center were found to satisfy the hierarchal structure [11]:

$$\frac{S_{p+2}(\tau)}{S_{p+1}(\tau)} = A_p \left[\frac{S_{p+1}(\tau)}{S_p(\tau)} \right]^\beta [S^{(\infty)}(\tau)]^{1-\beta}, \qquad (5)$$

where $S_p(\tau) \equiv \langle |v(t+\tau) - v(t)|^p \rangle$, with v being one of the three velocity components, A_p are numbers independent of τ, and $S^{(\infty)}(\tau) \equiv \lim_{p\to\infty} S_{p+1}(\tau)/S_p(\tau)$. The smaller the parameter β, the more intermittent the velocity fluctuation is. The two horizontal velocity components were found to be characterized by the same value of β but the vertical velocity component by a smaller value of β. This distinction was attributed [10] to the presence of the plumes which makes the vertical velocity component more intermittent. As a further test of our method, we check whether the horizontal and vertical components of $\mathbf{v_b}(t)$ at the cell center will now be characterized by the same value of β. We follow the procedure developed in Ref. [12] to calculate the relative scaling exponents $\rho(p)$ of the normalized structure functions, defined by $S_p(\tau)/S_2(\tau)^{p/2} \sim [S_1(\tau)/S_2(\tau)^{1/2}]^{\rho(p)}$. If Eq. (5) holds, we have [12] $\Delta\rho(p+1) = \beta\Delta\rho(p) - (1+\beta)$, where $\Delta\rho(p) \equiv \rho(p+1) - \rho(p)$. In Fig. 2, we plot $\Delta\rho(p+1)$ vs $\Delta\rho(p)$. We see that the data points for the three components of $\mathbf{v_b}(t)$ nearly fall on the same line, showing that the difference in the β-values of the horizontal and vertical components indeed becomes vanishingly small.

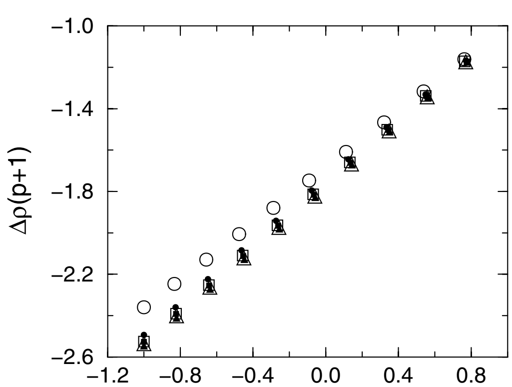

FIG. 2. $\Delta\rho(p+1)$ vs $\Delta\rho(p)$ for the x-component (triangles), y-component (squares), and z-component (circles) for $\mathbf{v}$ (open symbols) and $\mathbf{v_b}$ (filled symbols) at the cell center for Ra $= 4.8 \times 10^9$.

Then we proceed to study the temperature dependence of the plume velocity so extracted. As seen in Fig. 3, $v_{pz} = a\delta T$ at the cell center and $v_{pz} = b\sqrt{T - T_s}$ for $T > T_0$ ($>T_s$) near the sidewall for some constants a and b. The fitted value of T_s differs from T_0 by about 1%. We believe that this difference is caused by drifts in the mean temperature of the bulk fluid over time and that the measurements at the two locations were not taken at the same time. Hence, it is reasonable to conclude that $v_{pz} \propto \sqrt{\delta T}$ for $\delta T > 0$ near the sidewall.

We now understand the observed temperature dependence using the equations of motion. At the cell center, the mean velocity vanishes. By balancing the viscous term with the buoyancy term in Eq. (1), we have

$$\text{center}: \frac{\nu v_{pz}}{l_c^2} = g\alpha\delta T \qquad (6)$$

for some length scale l_c, which explains the temperature dependence observed. Since plumes are generated from the thermal boundary layer, we expect l_c to be of the order of the thermal boundary layer thickness λ_{th}. We calculate l_c from the fitted value of a and compare l_c/L with λ_{th}/L [13]. As shown in Fig. 4, l_c/L, after being scaled down by a factor of about 2, coincides with λ_{th}/L over the limited Ra range studied.

Near the sidewall, the average of the plume velocity is equal to the average vertical velocity, which is essentially the mean large-scale circulation velocity and is relatively large. Thus, the advection term should dominate over the viscous term. For $\delta T > 0$, we equate the advection term by the buoyancy term in Eq. (1) and get

$$\text{sidewall}: \frac{v_{pz}^2}{l_s} = g\alpha\delta T \quad \text{for } \delta T > 0 \qquad (7)$$

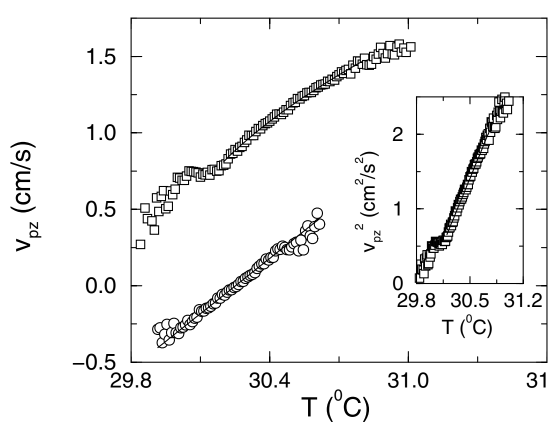

FIG. 3. v_{pz} vs T at the cell center (circles) and near the sidewall (squares) at Ra $= 2.6 \times 10^9$. The solid lines from bottom to top are, respectively, the fits of $a\delta T$ and $b\sqrt{T - T_s}$ for some constants a and b. The fitted value for T_s is 29.9 °C while T_0 is 30.2 °C. Inset shows v_{pz}^2 vs T for the near-sidewall data: the linear dependence on T is clearer.

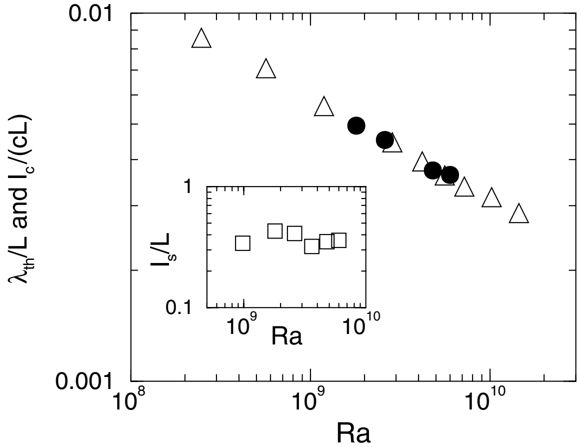

FIG. 4. λ_{th}/L (triangles) [13] and $l_c/(cL)$ (filled circles) vs Ra with $c = 1.9$. Inset shows l_s/L vs Ra.

for some other length scale l_s. Hence, $v_{pz} \propto \sqrt{\delta T}$ for $\delta T > 0$, reproducing the temperature dependence observed near the sidewall. Since the advection term is dominated by the mean large-scale circulation, it seems reasonable to expect l_s to be of the order of L. Indeed l_s, calculated from the fitted value of b, is about 0.3–0.4L and has little dependence on Ra (see inset of Fig. 4).

We further obtain the average local heat flux in the vertical direction at the cell center, $\langle j_z(t)\rangle_c$, which can be thought of as a pointwise Nusselt number at the cell center. Since $\langle \mathbf{j_b}(t)\rangle = 0$, $\langle j_z(t)\rangle = \langle j_{pz}(t)\rangle = \langle v_{pz}(t)\delta T(t)\rangle L/(\kappa\Delta)$. Using (6), we have

$$\langle j_z(t)\rangle_c = \mathrm{Ra}\frac{\langle(\delta T)^2\rangle_c}{\Delta^2}\left(\frac{l_c}{L}\right)^2. \tag{8}$$

Note that $\Delta_c \equiv \sqrt{\langle(\delta T)^2\rangle_c}$ is the root-mean-square temperature fluctuation at the cell center. It was found that $\Delta_c/\Delta \sim \mathrm{Ra}^{-1/7}$ both in low temperature helium gas [14] and in water [15]. As l_c/L goes like λ_{th}/L (see Fig. 4) and $\lambda_{th}/L \sim \mathrm{Ra}^{-2/7}$ [13], we have $\langle j_z(t)\rangle_c \sim \mathrm{Ra}^{1/7}$. We now show that one has to conclude that heat cannot be mainly transported through the central region of the convection cell based on this scaling behavior of $\langle j_z(t)\rangle_c$ with Ra. Suppose heat is mainly transported through the central region of the convection cell, then the Nusselt number (Nu) would be given by $\mathrm{Nu} \approx r\langle j_z(t)\rangle_c$, where r is the ratio of the cross-sectional area of the central region to the total cross-sectional area of the cell. Now r is approximately constant over the Ra range studied; thus one would get $\mathrm{Nu} \sim \mathrm{Ra}^{1/7}$. But this is clearly inconsistent with the experimental measurements of a scaling exponent of approximately 2/7 of Nu with Ra [16]. Hence, heat cannot be mainly transported through the central region but has to be transported through regions near the sidewalls of the convection cell.

In summary, we have presented a scheme to extract the velocity of buoyant structures in turbulent thermal convection. Our scheme involves a decomposition of the local velocity measurement into two parts. The part that is correlated with *some* function of the local temperature measured at the same time is taken as the velocity of these buoyant structures. If the velocity is uncorrelated with *any* function of the temperature, then such a part will not exist and no buoyant structures are identified. Applying this scheme to measurements taken at the center and near the sidewall of the convection cell where the dominant buoyant structures are plumes, we have found the temperature dependence of the plume velocity at these two locations and understood such dependence from the equations of motion. Moreover, we have obtained the average local heat flux in the vertical direction at the cell center, and found that it has a scaling dependence with Ra different from that of Nu. This difference leads to the conclusion that heat is not mainly transported through the central region but instead through regions near the sidewalls of the convection cell.

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[9] $\langle\langle \mathbf{v}|T(t)\rangle f(T)\rangle \equiv \int[\int \mathbf{v}P(\mathbf{v}|T)d\mathbf{v}]f(T)P(T)dT$ where $P(\mathbf{v}|T)$ is the conditional pdf of $\mathbf{v}$ on T and $P(T)$ is the pdf of T. Since $P(\mathbf{v}|T)P(T) = P(\mathbf{v}, T)$, the joint pdf of $\mathbf{v}$ and T, we have $\langle\langle \mathbf{v}|T(t)\rangle f(T)\rangle = \langle \mathbf{v}f(T)\rangle$.
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