EXPERIMENTAL STUDY OF RELATIVE VELOCITY FLUCTUATIONS IN TURBULENCE

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Photon correlation spectroscopy was used to explore turbulent pipe flow behind a grid. Measurements of the light intensity correlation function indicate that the probability distribution function of the relative velocity in turbulent flow is well approximated by a product of a lorentzian function and a gaussian-like function.

A quantity of fundamental interest in the theory of turbulence is the velocity difference, $V(R, t)$, between a pair of points in the turbulent fluid separated by a distance R. The statistical property of the velocity fluctuations $V(R, t)$ can be characterized by its moments, $\langle |V(R, t)|^n \rangle$, or more generally, by its probability distribution function, $P(V, R)$ [1,2]. We report here a light-scattering study of turbulent flow from which the functional form of $P(V, R)$ can be inferred.

It was shown [3–5] that the distribution function $P(V, R)$ is accessible by the technique of photon correlation spectroscopy (PCS). The correlation function of the light intensity, $I(t)$, scattered by small particles suspended in the turbulent fluid, $g(t) = \langle I(t')\, I(t'+t) \rangle$, has the form [3,4]

$$g(t) = 1 + G(qt, L)\,, \tag{1}$$

where

$$G(qt, L) = \int_0^L \mathrm{d}R\, h(R) \int_{-\infty}^{\infty} \mathrm{d}V P(V, R) \cos(qtV)\,. \tag{2}$$

In the above, the velocity difference is defined as

$$V(R, t) = v(r(t)) - v(r(t) + R)\,,$$

where $v(r(t))$ is the local velocity of the fluid, and V is the component of $V(R, t)$ along the scattering vector q. The scattering volume viewed by a photodetector is assumed to be quasi-one-dimensional with length L, and $h(R)$ is the number fraction of particle pairs separated by a distance R in the scattering volume. The scattering vector q has the amplitude $q = (4\pi/\lambda) \sin(\theta/2)$, where θ is the scattering angle, and λ is the wavelength of light in the fluid. Eqs. (1) and (2) say that the light scattered by each pair of particles contributes a phase factor $\cos(qtV)$ (due to the frequency beating) to the intensity correlation function, $g(t)$, and $g(t)$ is an incoherent sum of these ensemble averaged (or time averaged) phase factors over all the particle pairs in the scattering volume.

We have explored the PCS technique to study the turbulent pipe flow behind a grid at moderate Reynolds numbers [3]. The Reynolds number of the grid flow is defined as $\mathrm{Re} = UM/\nu$, where U is the mean flow velocity at the center line of the pipe, M the aperture size of the grid which generates turbulence, and ν is the kinematic viscosity of fluid. In this experiment water seeded with polystyrene spheres of diameter 0.06 microns, is circulated through a closed system by a pump. A section of the pipe, of 2.0 inch diameter, is made of glass to admit the incident laser beam and observe the scattering. Undesirable velocity fluctuations produced by the pump or by the pipe corners, are damped out by a screen (aperture size 2.0 mm) on the high-pressure side of the grid. The aperture size of the grid, M, was 3.1 mm. The measuring point was on the axis of the pipe and 28 cm downstream from the grid ($x/M = 90$). Measure-

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ments of $g(t)$, at room temperature, were performed with use of the standard light-scattering apparatus with a multi-channel correlator (Langley Ford 1096). The focused laser beam from a 1 W argon-ion laser enters the fluid along the axis of the pipe through an optical window. A lens focuses the scattered laser beam on a plane where a slit is located. The lens was placed in such position that the size of the scattered laser beam is the same as that of its image. It is the width of the slit which controls the size of the scattering volume viewed by the photomultiplier, located far behind the slit. The incident beam and the aligned optical apparatus (the lens, the slit and the photomultiplier) define the scattering angle θ. The photomultiplier output went to the correlator, whose output gives $g(t)$.

It was found in the experiment that when Re becomes larger than a transition Reynolds number, Re_c (~ 300–400), the function $G(qt, L)$ extracted from the measured $g(t)$ has the scaling form, $G(qt, L) = G(\kappa)$, where $\kappa = qt\bar{u}(L)$. Here $\bar{u}(L)$ is the characteristic turbulent velocity associated with eddies of size L. Both the functional form of $G(\kappa)$ and its scaling argument κ provide information about the statistical properties of the velocity fluctuations $V(R, t)$. Our measurements of $g(t)$ suggest that the distribution function $P(V, R)$ for small values of $V(R, t)$ is lorentzian-like when $\mathrm{Re} \gtrsim \mathrm{Re}_c$. Equivalently the characteristic function (the Fourier transform of $P(V, R)$) decays exponentially. By assuming

$$P(V, R) \sim \{1 + [V/\bar{u}(R)]^2\}^{-1}$$

($V = \boldsymbol{q} \cdot \boldsymbol{V}(R, t)/q$) and

$$h(R) \approx (2/L)(1 - R/L)$$

(scatterers are uniformly distributed), the integrals in eq. (2) can be carried out. The function $G(qt, L)$ turns out to be the incomplete gamma function with $qt\bar{u}(L)$ as its argument. This equation was extremely well fitted to the measured $g(t)$. An example of this good fit is seen in fig. 1 (solid line), which shows $g(t)$ at $\mathrm{Re} = 1.395$, $\theta = 90°$ and $L = 0.6$ mm. If the flow is assumed to be locally isotropic, this fitting suggests that the three dimensional distribution function $P(V, R)$ is lorentzian-square:

$$P(V, R) \sim \{1 + [|V|/\bar{u}(R)]^2\}^{-2} .$$

We also found that $\bar{u}(L)$ has a scaling form,

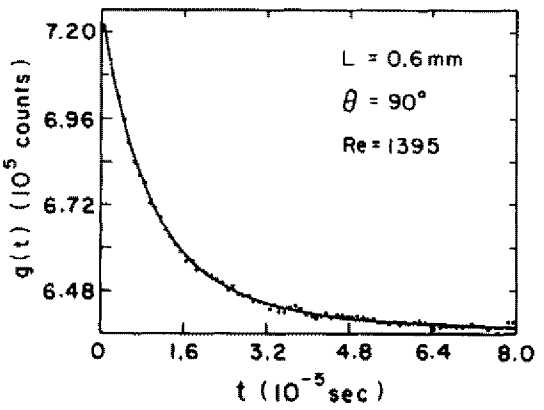

Fig. 1. A typical correlation function $g(t)$ versus t. The solid curve is a fit to the incomplete gamma function.

$\bar{u}(L) \sim L^{\zeta}$. The exponent ζ shows a nontrivial Re-dependence and reveals a transition character when Re is near and above Re_c. When $\mathrm{Re} \gtrsim 1400$, ζ has climbed to, and saturated at, a value close to 1/3 (the Kolmogorov value). Fig. 2 shows more clearly the variation of ζ with Re. The above features summarized in figs. 1 and 2 were also observed when both the grid and the fine screen were removed [6].

In theories of fully developed isotropic turbulence [7–9,2] $V(R, t)$ is expected to be self-similar, in that the statistical properties of $V(R, t)$ over varying length scales, R, become identical under an appropriate scaling of velocities (i.e. $\langle V(R, t)^n \rangle$ obeys a power law of R). It is easy to show that [1] the self-similar behavior of $V(R, t)$ can be obtained if its distribution function $P(V, R)$ is a homogeneous function $P(V/\bar{u}(R))$, where $\bar{u}(R)$ is a scaling velocity. Then eq. (2) becomes

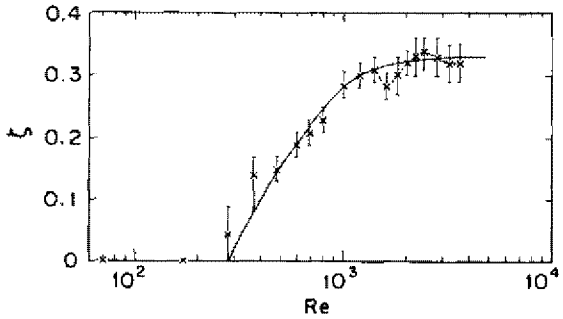

Fig. 2. The variation of the exponent ζ with the Reynolds number Re. The solid curve is drawn by eye through the data points. The dashed curve shows a small oscillation of ζ.

$$G(qt, L) = \int_0^L \mathrm{d}R\, h(R) F(qt\bar{u}(R)) , \qquad (3)$$

where $F(x)$ is the Fourier cosine transform of $P(V, R)$. In the scaling theories of turbulence, attention is focused on the so called "active region", where vorticity is highly localized (the intermittency effect), and the contributions from the "passive region" to the statistical properties of the turbulent fluctuations are completely neglected. Therefore it is believed that the higher moments of $V(R, t)$ are convergent and give information about the intermittent turbulent fluctuations. However, our experimental results indicate that for small relative velocity fluctuations, $P(V, R)$ is lorentzian-like (algebraic decay) whose moments higher than the first diverge. Of course $P(V, R)$ cannot remain lorentzian for very large velocity fluctuations, since the turbulent energy injection rate is finite.

Referring to the above experimental results, Onuki proposed [10] that the three dimensional distribution $P(V, R)$ for isotropic flow is a product of two functions. One function is associated with the large velocity fluctuations, having finite moments and being characterized by a scaling velocity $u(R)$. The other function is associated with the small velocity fluctuations, having lorentzian-square form (in the three dimensional case) and being characterized by a scaling velocity $\bar{u}(R)$ as mentioned above. Under the assumption that the moments $\langle |V(R, t)|^n \rangle$ obey the original Kolmogorov theory [7], it is proved by Onuki that the scaling velocities $u(R)$ and $\bar{u}(R)$ have the same R-dependence. Onuki also predicts that the characteristic function F should cross over from exponential-like form ($F \sim 1 - \bar{u}(R)qt$) to gaussian-like form ($F \sim 1 - \text{const} \times u(R)\bar{u}(R)(qt)^2$) when $qtu(R) \sim 1$.

If $P(V, R)$ is a product of two functions, the Fourier transform of $P(V, R)$ is the convolution of the two individual Fourier transforms. Because the distribution function $P(V, R)$ is assumed to be a product of $\{1 + [V/\bar{u}(R)]^2\}^{-1}$ (lorentzian) and $\exp\{-[V/\sqrt{2}\,u(R)]^2\}$ (gaussian), then in the small t limit eq. (3) becomes

$$G(qt, L) \approx a_1 [\mathrm{e}^{-t/a_2} \operatorname{erfc}(-t/a_3 + a_3/2a_2) + \mathrm{e}^{t/a_2} \operatorname{erfc}(t/a_3 + a_3/2a_2)] , \qquad (4)$$

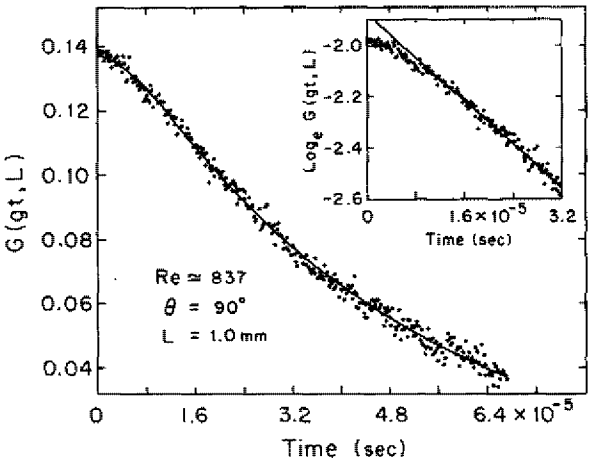

Fig. 3. A typical plot of the function $G(qt, L)$ versus t. The solid curve is a fit to eq. (4). The inset is a semilog plot of $G(qt, L)$ versus t at the same parameters. The solid line in the inset shows the exponential behavior of $G(qt, L)$ away from $t=0$.

where $\operatorname{erfc}(x)$ is the complementary error function, $a_2 = 1/0.643q\bar{u}(L)$, and $a_3 = \sqrt{2}/0.643qu(L)$. To get eq. (4) we used the gaussian quadrature method [11] to evaluate the integral in eq. (3), i.e. the integral in eq. (3) is approximated by the characteristic function $F(qt\bar{u}(R))$ evaluated at its zero-order abscissa $0.643qt\bar{u}(L)$. It turns out that the error introduced by this approximation is no more than 13% when $qt\bar{u}(R) \leqslant 1$. Eq. (4) was well fitted to the measured $G(qt, L)$ in the short time region, where a_1, a_2, and a_3 are fitting parameters. A typical fit is displayed in fig. 3 (solid line) which shows $G(qt, L)$ at $\mathrm{Re} \approx 837$, $\theta = 90°$, and $L = 1.0$ mm. The fitting results are $a_2 = 4.77 \times 10^{-5}$ s and $a_3 = 8.30 \times 10^{-6}$ s.

From the fitting in fig. 3 it was found that the ratio of the two decay times, a_2/a_3, is 5.75. Because the gaussian factor in $P(V, R)$ introduces a round-off to $G(qt, L)$ near $t=0$, the absolute value of the slope of $G(qt, L)$ should decrease as $t \to 0$. This decrease in slope is more clearly seen in the semilog plot of $G(qt, L)$ in the inset of fig. 3. The inset of fig. 3 also shows that $G(qt, L)$, away from $t=0$, is best fitted to a single exponential which is associated with the lorentzian factor in $P(V, R)$. Many measured $G(qt, L)$ at various slit widths and Re were fitted to eq. (4). It was found that the typical value of the ratio, $\gamma = u(L)/\bar{u}(L) = \sqrt{2}\,a_2/a_3$, is about 7, and does not change very much with Re and L. Using the fitted

values of a_2 and a_3 we calculated that $u(L)/U \approx 4.0\%$, and $\sqrt{\langle |V|^2 \rangle}/U \approx 2.7\%$. Whereas the turbulent intensity measured by laser Doppler velocimetry, $\sqrt{\langle \delta v^2 \rangle}/U$, was about 5.6%, where δv is the fluctuation part of the local velocity.

Kraichnan pointed out [12] that the brownian motion of the seed particles may affect the behavior of $G(qt, L)$ near $t=0$. The effect of the brownian motion is to contribute a factor $\exp(-2Dq^2t)$ to $G(qt, L)$, where D is the diffusion constant of brownian particles. The diffusion time $T_D = (2Dq^2)^{-1}$ can be obtained by measuring the correlation function $g(t)$ when the flow is absent [13]. In our experiment the measured diffusion time $T_D = 1.13 \times 10^{-4}$ s. By equating the brownian motion contribution with that of the gaussian round-off of $G(qt, L)$ [10] in the short-time region, we can show that the brownian motion will eventually dominate the gaussian round-off effect when $t < 3\pi D/u(L)\bar{u}(L)$. This time is 6.83×10^{-6} s for fig. 3 and 8.9×10^{-7} s for fig. 1. Because the time resolution (the smallest sampling time) is 10^{-6} s in fig. 1, the brownian effect can be neglected in this case. For fig. 3 the time resolution of the measurement of $G(qt, L)$ is 10^{-7} s, so that the brownian motion must be taken into account. The measured $G(qt, L)$ shown in fig. 3 has been divided by $\exp(-t/T_d)$ to eliminate the brownian motion effect.

In conclusion, our measurements suggest that the distribution function $P(V, R)$ changes its functional form lorentzian to gaussian-like with increasing $V(R, t)$, and hence must be characterized by at least two parameters $\bar{u}(R)$ and $u(R)$. It was also shown that the function $G(qt, L)$ is a homogeneous function of $qt\bar{u}(L)$, where $\bar{u}(L)$ is the characteristic velocity associated with the small velocity fluctuations. The L-dependence of $\bar{u}(L)$ indicates that the small velocity fluctuations possess similar scaling character as that in the active region. Our finding that $P(V, R)$ is adequately represented by the product of lorentzian and gaussian factors, is consistent with the notion that $V(R, t)$ arises from two distinct regions of turbulent fluid [10].

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