Measured Local-Velocity Fluctuations in Turbulent Convection

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Turbulent Rayleigh-Bénard convection in water has been studied using a newly developed technique of incoherent cross-correlation spectroscopy. At high Rayleigh number Ra (hard turbulence), local velocity fluctuations are isotropic, and their probability density functions in the center region have an invariant Gaussian form, with the rms velocity $v_0 \sim \mathrm{Ra}^{0.44}$. At low Ra in the soft turbulence regime, the probability density functions for the vertical velocity fluctuations do not have a universal form and appear to depend on the coherence of thermal plumes emitted from the boundary layers.

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The discovery of scaling laws in the heat flux and temperature statistics in turbulent convection [1–3] has stimulated considerable experimental [4,5] and theoretical [6] efforts, aimed at explaining the observed scaling laws in the temperature field. The theoretical calculations arrive at similar conclusions for the temperature field, but have different assumptions and predictions for the velocity field in turbulent bulk regions and near viscous and thermal boundary layers. Direct measurements of the velocity field, therefore, become important to verify assumptions and test predictions of various theoretical models [6,7]. In contrast to the great number of temperature measurements, however, experimental information about velocity fluctuations and their statistics in turbulent convection is limited. The lack of velocity information is due partially to the fact that the conventional methods for measuring velocity, such as hot-wire anemometry and laser Doppler velocimetry (LDV), are not suitable for thermal turbulence. Strong temperature fluctuations in convective turbulence can ruin the calibration of a hot-wire anemometer. Fluctuations of the fluid refractive index due to large temperature fluctuations may cause the two laser beams used in LDV to wander and defocus in the fluid. This corruption of laser beam properties will reduce the signal-to-noise level of LDV.

In this Letter we report a light-scattering study of turbulent convection in water with the Rayleigh number Ra ranging from 10^7 to 10^{11}. A newly developed technique of incoherent cross-correlation spectroscopy (ICS) [8] is used to measure the local velocity v and its probability density function (PDF) $P(v)$ in the center region of the convection cell. Previous temperature measurements have revealed two distinct turbulent states in convection: soft turbulence when $\mathrm{Ra} < 10^8$ and hard turbulence for $\mathrm{Ra} > 10^8$ [1]. Our velocity measurements show that local velocity fluctuations in the hard turbulence regime are isotropic, and their PDF's have an invariant Gaussian form $P(v) = (1/v_0)\exp[-(v/v_0)^2]$. The rms velocity, $v_0 \sim \mathrm{Ra}^{0.44}$, is in good agreement with the theoretical predictions [2,9,10]. In the soft turbulence regime, on the other hand, velocity fluctuations in the vertical direction parallel to gravity differ substantially from those in the horizontal direction. The PDF's for the vertical velocity fluctuations do not have a universal form and appear to depend on the coherence of the thermal plumes emitted from the boundary layers.

In the experiment (to be described below) a laser beam from an argon-ion laser traverses through the convecting fluid. The laser is under multiline operation, and the beam consists of light with a wavelength range from 457.9 to 514.5 nm. The fluid is seeded with small, neutrally buoyant polymer latex spheres, which scatter light and follow the local flow. Two photodetectors are used to record the two strongest scattering intensities, $I_b(t)$ and $I_g(t)$, of the blue and green lights, respectively. The velocity of the seed particles is determined by measuring the time of flight for the particles to cross the laser beam with a known waist radius σ. With the ICS scheme, this transit time is obtained from the intensity cross-correlation function

$$g_c(t) = \frac{\langle I_b(t' + t) I_g(t') \rangle}{\langle I_b \rangle \langle I_g \rangle} = 1 + b G_c(t), \qquad (1)$$

where b (≤ 1) is an instrumental constant. Because there is no phase coherence between $I_b(t)$ and $I_g(t)$, $g_c(t)$ is only sensitive to the scattering amplitude fluctuations produced by the seed particles moving in and out of the laser beam. This method adopts the advantages of a laser source while eluding its coherence, which is responsible for the rapid phase fluctuations of the scattered light produced by the Doppler effect.

When the seed particles have a uniform velocity v, $G_c(t)$ in Eq. (1) takes the form [8]

$$G_c(t) = \frac{1}{N} e^{-(vt/\sigma)^2}, \qquad (2)$$

where N is the number of particles in the scattering volume. In obtaining Eq. (2), the laser beam profile has been assumed to have a Gaussian form $I(r) = I_0 \exp[-2(r/\sigma)^2]$, with r being the radial distance from

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the center of the laser beam and I_0 the light intensity at the beam center. For turbulent flows, $G_c(t)$ becomes

$$G_c(t) = \frac{1}{N}\int_{-\infty}^{\infty} dv_i \int_{-\infty}^{\infty} dv_j\, P_2(v_i, v_j)\, e^{-(v_i^2+v_j^2)t^2/\sigma^2}, \tag{3}$$

where $P_2(v_i, v_j)$ is the PDF for the velocity components v_i and v_j in two different directions, both orthogonal to the incident laser beam. By changing the direction of the incident laser beam, one can measure different components of the local velocity. Because of the axial symmetry of the laser beam, $G_c(t)$ is only sensitive to $v_i^2 + v_j^2$. This feature is particularly useful in studying the convective flow in the center region, where the mean flow velocity is zero [7] and, thus, velocity fluctuations can be directly measured. Because the acceptance angle of the receiving optics is large enough, small amplitude beam wandering in the convecting fluid will not affect the measurement of $g_c(t)$.

The convection cell used in the experiment was a vertical cylindrical cell with inner diameter 20 cm. The upper and lower plates were made of brass and their surfaces were gold plated. The sidewall of the cell was a cylindrical ring made of transparent Plexiglas to admit the incident light and transmit the scattered light. Three cylindrical rings with heights of 6.6, 20, and 40 cm were used, respectively, to extend the accessible range of Ra. The corresponding aspect ratios (A = diameter/height) for these cells are 3.0, 1.0, and 0.5. The temperature of the upper plate was regulated by passing cold water through the cooling chamber fitted on the top of the plate. The lower plate was heated uniformly at a constant rate with an electric heating film embedded on the backside of the plate. The temperature difference ΔT between the two plates was measured by two thermistors embedded in the two plates. The control parameter in the experiment is the Rayleigh number Ra $= \alpha g h^3 \Delta T/\nu\kappa$, with α being the thermal expansion coefficient, g the gravitational acceleration, h the height of the convection cell, and ν and κ the kinematic viscosity and the thermal diffusivity of the fluid, respectively.

The cell was filled with distilled water, which was seeded with monodispersed polymer latex spheres of 0.95 μm in diameter. These particles had a high surface charge density to ensure their stability in the convecting fluid. The volume fraction of the seed particles was $\sim 5 \times 10^{-7}$. Measurements of $g_c(t)$ were performed with a standard light-scattering apparatus and an ALV-5000 digital correlator. A focused incident beam from an argon-ion laser traversed through the center of the cell. A lens was placed at 90° with respect to the incident direction, and projected the scattered laser beam in the flow cell onto an adjustable slit with a 1:1 magnification. Light passing through the slit fell on two photomultipliers, which recorded the time-varying intensities $I_b(t)$ and $I_g(t)$, respectively. The two photomultipliers were mounted at a right angle on a cubic box, in which a beam splitter was placed at the center. An interference filter was placed in front of each of the photomultipliers. The output signals from the two photomultipliers were fed to an ALV-5000 correlator, whose output gives $g_c(t)$. Each $g_c(t)$ was measured for more than 20 min to ensure that the time average included all the flow configurations. This long-time average is essential in order to obtain an accurate rms velocity [11]. The laser beam profile $I(r)$ was measured using a micrometer controlled translational stage, a small pinhole (5 μm in diameter), and a photodiode. It is found that the measured $I(r)$ is, indeed, of a Gaussian form with the $1/e^2$ beam radius $\sigma = 48 \pm 3$ μm.

Figure 1 shows the measured $g_c(t)$ as a function of the delay time t at Ra $= 3.5 \times 10^{10}$. The measurement was made when the laser beam was horizontally shone through the center of the cell with $A = 0.5$. The data can be well fitted by the function $1 + a/[1 + (\Gamma t)^2]$ with $a = 0.045$ and $\Gamma = 0.15$ ms^{-1} (the solid curve). The Lorentzian form of the measured $G_c(t)$ [$= g_c(t) - 1$] differs substantially from the Gaussian function (the dashed curve) shown in Eq. (2), which has been measured in a laminar Poiseuille flow [8]. According to Eq. (3), the PDF, $P_2(v_i, v_j)$, can be obtained by a simple Laplace inversion of the measured $G_c(t)$. For a Lorentzian $G_c(t)$, we find the corresponding $P_2(v_i, v_j)$ is of the Gaussian form

$$P_2(v_i, v_j) = \frac{1}{v_0^2}\, e^{-(v_i^2+v_j^2)/v_0^2}, \tag{4}$$

where $v_0 = \Gamma\sigma$ is the rms velocity. To obtain Eq. (4), we have used the fact that the velocity fluctuations in the center region are isotropic [see Fig. 3(a) below].

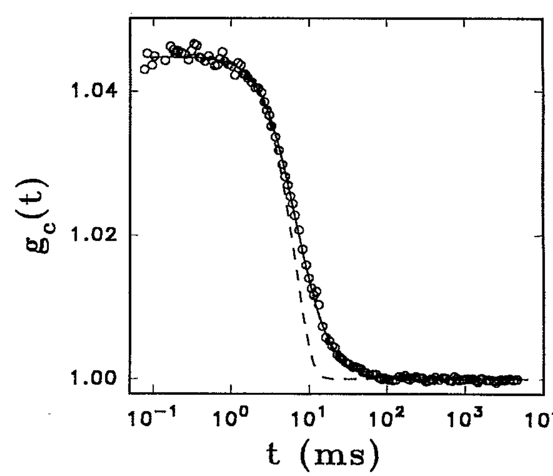

FIG. 1. The measured $g_c(t)$ as a function of the delay time t at Ra $= 3.5 \times 10^{10}$. The solid curve is a fit to $1 + a/[1 + (\Gamma t)^2]$ with $a = 0.045$ and $\Gamma = 0.15$ ms^{-1}. The dashed curve shows a Gaussian function $1 + a\exp[-(\Gamma t)^2]$.

Gaussian-like velocity PDF's have also been found in many barotropic turbulent flows [12].

The Lorentzian form of the measured $G_c(t)$ is found to remain unchanged when Ra is in the range between 10^8 and 10^{11}. In this Ra range only the decay rate Γ changes with Ra. Plots of $G_c(t)$ at different Ra can be brought into coincidence by scaling the time axis with Γ. Figure 2 shows typical $G_c(t)$ as a function of Γt for various values of Ra when the laser beam was (a) horizontally and (b) vertically shone through the cell center. Note that the two beam orientations actually probe different components of the local velocity. In the former case (horizontal beam) the velocity components in the vertical direction and in one of the horizontal directions are measured, whereas in the latter case (vertical beam) only the horizontal components of the local velocity are measured. It is seen from Fig. 2 that the measured $G_c(t)$'s in the hard turbulence regime superimpose with each other. Furthermore, the measured $G_c(t)$'s in two different beam orientations are found to have the same Lorentzian form. This suggests that the functional form of $P_2(v_i, v_j)$ and, hence, the turbulent structure in the hard turbulence regime are invariant with Ra. In mixing length theories, velocity fluctuations are often related to temperature fluctuations δT through the buoyancy effect: $v(t) = [\alpha g h \delta T(t)]^{1/2}$ [2]. If this is true in a strict deterministic sense (i.e., hotter fluid moves faster), then the PDF for v would be the same as that for $\sqrt{\delta T}$. In fact, it is known that the PDF $P(\sqrt{\delta T})$ [$= 2\sqrt{\delta T}\,\tilde{P}(\delta T)$] for $\sqrt{\delta T}$ is not a Gaussian function, because the measured PDF $\tilde{P}(\delta T)$ for δT is a simple exponential function [1,2]. Figures 1 and 2 thus imply that velocity fluctuations at small scales are not strongly influenced by the buoyancy that drives large-scale motions.

With the measured σ and Γ, we now plot the rms velocity v_0 ($= \Gamma\sigma$) as a function of Ra. Figure 3(a) compares the measured v_0 for the vertical beam (open circles) with that for the horizontal beam (solid circles) at different Ra. The measurements were made in the cell with $A = 0.5$. It is found from Figs. 2 and 3(a) that velocity fluctuations in different directions do not just have the same Gaussian PDF, but also have the same rms velocity. Therefore, the velocity fluctuations in the center region are isotropic. As shown in Fig. 3(b), the measured v_0 is well described by the power law $v_0 = 2.2 \times 10^{-5}\mathrm{Ra}^{\varepsilon}$ (the solid line) with the exponent $\varepsilon = 0.44 \pm 0.015$. If the Peclet number Pe ($= v_0 h/\kappa$) is chosen as a dimensionless velocity, we find from Fig. 3(b) that $\mathrm{Pe} = 0.30\mathrm{Ra}^{0.44}$. Using mixing length ideas, Kraichnan [10] predicts $\varepsilon = 4/9$, and recent scaling arguments [2,9] give $\varepsilon = 3/7$. While we cannot resolve the small difference between the two predicted values of ε within the experimental uncertainties, the measured ε is certainly in good agreement with the theoretical predictions. A similar value of ε was also obtained in previous experiments [11,13] where the velocity data from different convection systems were compiled together. However, difficulties in measuring small velocity fluctuations and limited statistics were reported in these measurements [11,13]. Our new scattering technique used in this experiment, on the other hand, is capable of measuring the velocity PDF with a large dynamic range and a high statistical accuracy.

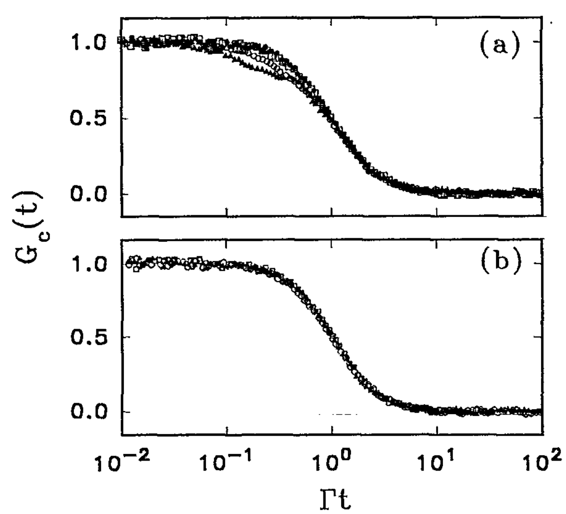

FIG. 2. The measured $G_c(t)$ vs Γt when the laser beam was (a) horizontally and (b) vertically shone through the cell center. The experimental conditions are (a) $\mathrm{Ra} = 7.5 \times 10^{10}$ (solid circles), 2.3×10^9 (open squares), 4.6×10^7 (open circles), and 1.4×10^7 (solid triangles); (b) $\mathrm{Ra} = 4.4 \times 10^{10}$ (open circles), 7.8×10^8 (solid triangles), and 2.1×10^7 (open squares).

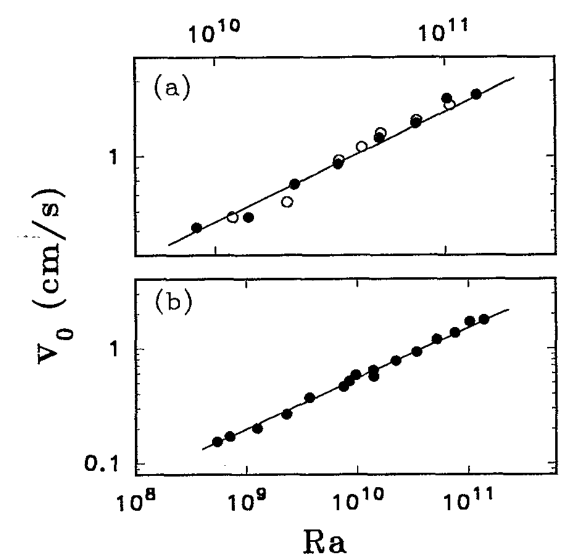

FIG. 3. (a) The measured v_0 vs Ra for the vertical beam (open circles) and for the horizontal beam (solid circles). (b) The measured v_0 as a function of Ra when the laser beam was horizontally shone through the center of the cells, with $A = 1.0$ and $A = 0.5$. The solid lines are the power law fit $v_0 = 2.2 \times 10^{-5}\mathrm{Ra}^{0.44}$ to the solid circles.

The above velocity measurements reveal that, in the hard turbulence regime, velocity fluctuations are isotropic and their PDF has an invariant Gaussian form. These findings are consistent with visual observations of the temperature field that, in the hard turbulence regime, thermal plumes emitted from the boundary layers are broken into smaller structures, as a result of strong turbulent mixing, before traversing through the center region of the cell [4]. An important question one might ask is how the local velocity fluctuates in the soft turbulence regime, where the thermal plumes are found to span the full height of the convection cell [4]. To answer this question, we now examine the measured $G_c(t)$ in the cell with $A = 3$. The shorter cell was used to reduce Ra and to allow the thermal plumes to traverse through the center region easily. As shown in Fig. 2(b), when the laser beam is vertically shone through the cell center, the measured $G_c(t)$ in the soft turbulence regime has the same Lorentzian form as that measured in the hard turbulence regime. When the laser beam is horizontally shone through the center region, however, the measured $G_c(t)$ continuously changes its functional form as Ra is increased from 1.2×10^7 to 2.0×10^8 [see Fig. 2(a)]. At lower Ra the measured $G_c(t)$ has a Lorentzian tail at large t, but its initial decay is slower than a Lorentzian function. As Ra is increased, the initial part of $G_c(t)$ approaches the Lorentzian form.

To characterize the decay of a non-Lorentzian $G_c(t)$, we measure the half-decay time $T_{1/2}$ of $G_c(t)$. For a Lorentzian function, the half-decay time $T_{1/2} = 1/\Gamma$. Figure 4 compares the measured v_0 $(= \sigma/T_{1/2})$ for the horizontal beam (solid circles) with that for the vertical beam (open circles) at different Ra. The solid line in Fig. 4 is an attempted power law fit $v_0 = 1.1 \times 10^{-5}\text{Ra}^{0.58}$ to the open circles, and the dashed line is an attempted power law fit $v_0 = 1.0 \times 10^{-4}\text{Ra}^{0.44}$ to the solid circles. Note that the fitted power law for the open circles differs from that obtained in the hard turbulence regime. Furthermore, the two power law fits in Fig. 4 are different from each other, both in amplitude and in exponent. Figure 4 thus suggests that, in the soft turbulence regime, velocity fluctuations in the vertical direction parallel to gravity differ substantially from those in the horizontal directions. From the measurements in the cell with $A = 3$, we conclude that in the soft turbulence regime the PDF's for the vertical velocity fluctuations do not have a universal form and appear to depend on the coherence of thermal plumes emitted from the boundary layers.

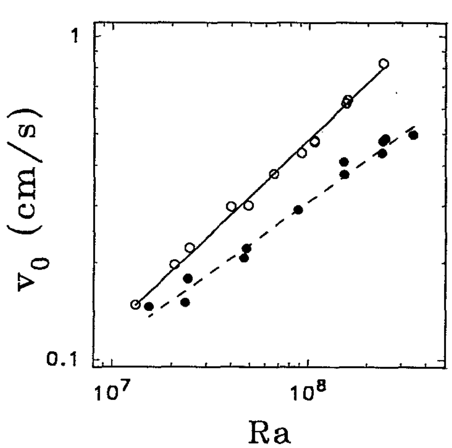

FIG. 4. The measured v_0 vs Ra for the vertical beam (open circles) and for the horizontal beam (solid circles) in the cell with $A = 3$. The solid line is an attempted power law fit $v_0 = 1.1 \times 10^{-5}\text{Ra}^{0.58}$ to the open circles, and the dashed line is an attempted power law fit $v_0 = 1.0 \times 10^{-4}\text{Ra}^{0.44}$ to the solid circles.

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