

# Velocity and temperature cross-scaling in turbulent thermal convection

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**Abstract.** We analyse simultaneous velocity and temperature measurements in turbulent thermal convection. Our results show the existence of a cross-scaling between the normalized velocity and temperature structure functions, as implied by the Bolgiano–Obukhov scaling, at the centre of the convection cell. We find that the cross-scaling exponents are, however, different from the values implied by the Bolgiano–Obukhov scaling.

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**1. Introduction**

The Rayleigh–Bénard convection system consists of a closed cell of fluid, which is heated from below and cooled on top. When the applied temperature difference exceeds some threshold, the fluid moves and convection occurs. The equations of motion, in Boussinesq approximation, are [1]

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \vec{\nabla} \vec{V} = -\vec{\nabla} p + \nu \nabla^2 \vec{V} + g \alpha \delta T \hat{z}, \quad (1)$$

$$\frac{\partial T}{\partial t} + \vec{V} \cdot \vec{\nabla} T = \kappa \nabla^2 T, \quad (2)$$

$$\vec{\nabla} \cdot \vec{V} = 0, \quad (3)$$

where  $\vec{V}$  is the velocity field,  $p$  the pressure divided by density,  $T$  the temperature field and  $\hat{z}$  is the unit vector in the vertical direction. Furthermore,  $\delta T = T - T_0$ , where  $T_0$  is the mean temperature of the bulk fluid,  $g$  is the acceleration due to gravity and  $\alpha$ ,  $\nu$  and  $\kappa$  are respectively the volume expansion coefficient, kinematic viscosity and thermal diffusivity of the fluid. The state of the fluid motion is characterized by the geometry of the cell and two dimensionless control parameters: the Rayleigh number,  $Ra = \alpha g \Delta L^3 / (\nu \kappa)$ , which measures how much the fluid is driven and the Prandtl number,  $Pr = \nu / \kappa$ , which is the ratio of the diffusivities of momentum and heat of the fluid. Here  $\Delta$  is the maintained temperature difference across the cell of height  $L$ . When  $Ra$  is sufficiently large, the convective motion becomes turbulent, and both the velocity and temperature fields exhibit complex fluctuations in time and space.

As in the study of other turbulent flows, a key issue is to make sense of these fluctuations. Conventionally, one is interested in the velocity and temperature structure functions,  $\tilde{S}_p(r)$  and  $\tilde{R}_p(r)$ :

$$\tilde{S}_p(r) \equiv \langle |V_r|^p \rangle, \quad (4)$$

$$\tilde{R}_p(r) \equiv \langle |T_r|^p \rangle. \quad (5)$$

Here  $\langle \dots \rangle$  denotes an ensemble average and  $V_r$  and  $T_r$  are respectively the velocity and temperature differences across a separation  $r$ :

$$V_r \equiv V(\vec{x} + \vec{r}, t) - V(\vec{x}, t), \quad (6)$$

$$T_r \equiv T(\vec{x} + \vec{r}, t) - T(\vec{x}, t). \quad (7)$$

In turbulent convection, there is an interplay between the velocity and temperature fields due to buoyancy. The applied temperature difference drives the fluid into motion and the temperature field is in turn advected by the velocity field. In the original work of Bolgiano [2] and Obukhov [3] for stably stratified turbulence (see [4] for a review), the vertical velocity and temperature power spectra, which are related to the Fourier transform of  $\tilde{S}_2(r)$  and  $\tilde{R}_2(r)$ , were predicted to have a scaling of  $k^{-11/5}$  and  $k^{-7/5}$  respectively in the Bolgiano regime, when the wavenumber  $k$  satisfies  $2\pi/k \geq l_B$ . Here,  $l_B \equiv \epsilon^{5/4}/[\chi^{3/4}(\alpha g)^{3/2}]$  is known as the Bolgiano scale [4], where  $\epsilon$  and  $\chi$  are the average energy and temperature dissipation rates. We shall see in section 3 that the Bolgiano scale can be understood as the length scale above which the power injected into the flow due to buoyancy is larger than the average energy dissipation rate. That is, the Bolgiano scale can be taken as the scale above which buoyancy is relevant. One can also define a local version of the Bolgiano scale using instead the locally averaged energy and temperature dissipation rates. For example, using the energy and temperature dissipation rates averaged over the horizontal plane at a given height of the cell, a  $z$ -dependent Bolgiano scale  $L_B(z)$  can be defined [5]. One expects that  $l_B \approx L_B(z)$  when  $z$  is in the central region of the convection cell. Thus, we shall take  $l_B$  to be the buoyancy-relevant scale at the cell centre. More recent theoretical works [6]–[9] also proposed this Bolgiano–Obukhov (BO) scaling in the Bolgiano regime for the unstably stratified case of Rayleigh–Bénard convection.

The spatial spectra or the related structure functions  $\tilde{S}_2(r)$  and  $\tilde{R}_2(r)$  are difficult to measure experimentally. Instead, it is common to measure their temporal counterparts,  $S_p(\tau)$  and  $R_p(\tau)$ :

$$S_p(\tau) \equiv \langle |V_\tau|^p \rangle, \quad (8)$$

$$R_p(\tau) \equiv \langle |T_\tau|^p \rangle, \quad (9)$$

and the corresponding frequency power spectra. Here  $V_\tau$  and  $T_\tau$  are the velocity and temperature differences between a time interval  $\tau$ :

$$V_\tau \equiv V(\vec{x}, t + \tau) - V(\vec{x}, t), \quad (10)$$

$$T_\tau \equiv T(\vec{x}, t + \tau) - T(\vec{x}, t). \quad (11)$$

In the evaluation of  $S_p$  and  $R_p$ , time averaging is used.  $S_p(\tau)$  and  $R_p(\tau)$  might be related to  $\tilde{S}_p(r)$  and  $\tilde{R}_p(r)$  by  $r = U\tau$ , where  $U$  is the mean velocity of the flow using Taylor hypothesis [10]. When the mean velocity vanishes, it was argued that [7] the wavenumber and frequency power spectra are related by a sweeping relation with  $\omega = ku_{rms}$ , where  $\omega$  is the frequency and  $u_{rms}$  is the root-mean-squared velocity fluctuation.

There were experimental reports that the vertical velocity and temperature frequency power spectra exhibit scaling in  $\omega$  that is consistent with the BO scaling [11]–[16]. These velocity and temperature measurements were taken at different locations in the cell. The velocity spectra were taken at about  $L/4$  from the bottom plate [11] and near the sidewall [12], whereas the temperature spectra were mostly taken at the cell centre [11, 13, 15, 16]. Thus, it is not obvious

that the vertical velocity and temperature power spectra exhibit scaling that is consistent with BO *together* and over the *same* frequency range.

Velocity measurements are difficult to measure in turbulent thermal convection. Before experimental velocity measurements were available, Ching [17] has used numerical velocity measurements [18] and experimental temperature measurements [19] to check the balance between thermal forcing and nonlinear velocity advection, which is a key ingredient leading to the BO scaling, and found that the balance does not hold.

In the present paper, we analyse the velocity and temperature measurements in turbulent thermal convection that were taken simultaneously and at the same location. The experiment was carried out in an aspect-ratio-one cylindrical cell of height  $L = 20.5$  cm and filled with water. The velocity was measured using a two-component laser Doppler velocimetry (LDV) system [20]. The temperature was measured using a small movable thermistor of 0.2 mm in diameter, time constant of 15 ms and temperature sensitivity of  $20 \text{ mK } \Omega^{-1}$ . The velocity and temperature measurements were taken simultaneously using a multichannel LDV interface module to synchronize the data acquisition [21]. In the present work, we study measurements taken at several locations inside the convection cell: the cell centre, near the sidewall (on the mid-plane of the cell at 8 mm from the sidewall) and near the bottom plate (1.5 cm above the centre of the bottom plate). The number of data points used in the analysis is 805235 at the cell centre, 438863 near the sidewall and 749563 near the bottom plate.

We first study the correlation between the velocity and temperature differences. Our results, presented in section 2, support that buoyancy is relevant to the statistical characteristics of the flow only in the Bolgiano regime. In section 3, we show that the BO scaling implies a cross-scaling between the normalized vertical velocity and temperature structure functions. In section 4, we demonstrate the existence of such a velocity and temperature cross-scaling in the Bolgiano regime at the cell centre. However, we find that the scaling exponents are different from those implied by the BO scaling. Finally, we give a summary in section 5.

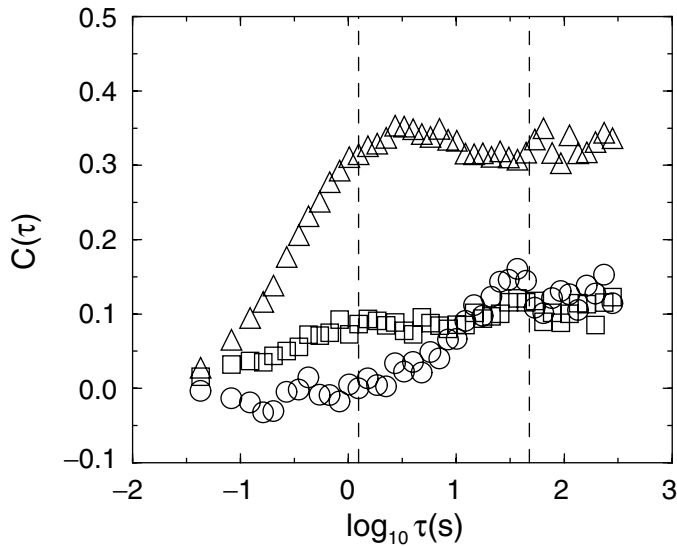
## 2. Correlation between the velocity and temperature differences

When buoyancy is important, one expects the vertical velocity fluctuation to have a positive correlation with the temperature fluctuation. To investigate the length scales over which the effect of buoyancy is significant, we study the correlation between the velocity and temperature differences  $V_\tau$  and  $T_\tau$  as a function of the time separation  $\tau$ . For this purpose, we calculate the correlation coefficient  $C(\tau)$ , defined as

$$C(\tau) \equiv \frac{\langle V_\tau T_\tau \rangle}{\langle V_\tau^2 \rangle^{1/2} \langle T_\tau^2 \rangle^{1/2}}. \quad (12)$$

If  $V_\tau$  and  $T_\tau$  are uncorrelated,  $C(\tau) = 0$ , and if there is a linear relation between  $V_\tau$  and  $T_\tau$ , then  $C(\tau) = 1$ . Besides the vertical velocity along the  $z$ -direction,  $C(\tau)$  is also calculated for velocity components along the  $x$  and  $y$  directions. The direction of the mean large-scale flow observed near the bottom plate defines the  $x$ -direction, and the  $x, y$  and  $z$  axes form a right-handed co-ordinate system.

Before presenting the results, we shall first discuss some important time scales in the problem. A well-defined oscillation has been observed in the velocity power spectrum [22]. This oscillation time  $\tau_0$  coincides with the decorrelation time in the temperature structure function  $R_2(\tau)$  [23]. Thus, we associate  $\tau_0$  with the largest length scale  $L$ . We then define the time scale  $\tau_B$  corresponding to the Bolgiano scale  $l_B$  by  $\tau_B \equiv \tau_0 l_B / L$  [17]. The average energy and



**Figure 1.** Correlation coefficient  $C(\tau)$  of the temperature difference with the  $x$  component ( $\circ$ - $\circ$ ),  $y$  component ( $\square$ - $\square$ ) and  $z$  component ( $\triangle$ - $\triangle$ ) of the velocity difference at the cell centre. The two broken lines show  $\tau_B$  and  $\tau_0$ .

temperature dissipation rates,  $\epsilon$  and  $\chi$ , can be found [24] in terms of  $Ra, Pr, \kappa, \Delta, L$  and the Nusselt number ( $Nu$ ), which is the heat flux normalized to the heat flux when there was only conduction. As a result,  $l_B$  can be expressed [25] in terms of  $Ra, Pr, L$  and  $Nu$ . Thus,  $\tau_B$  is given by

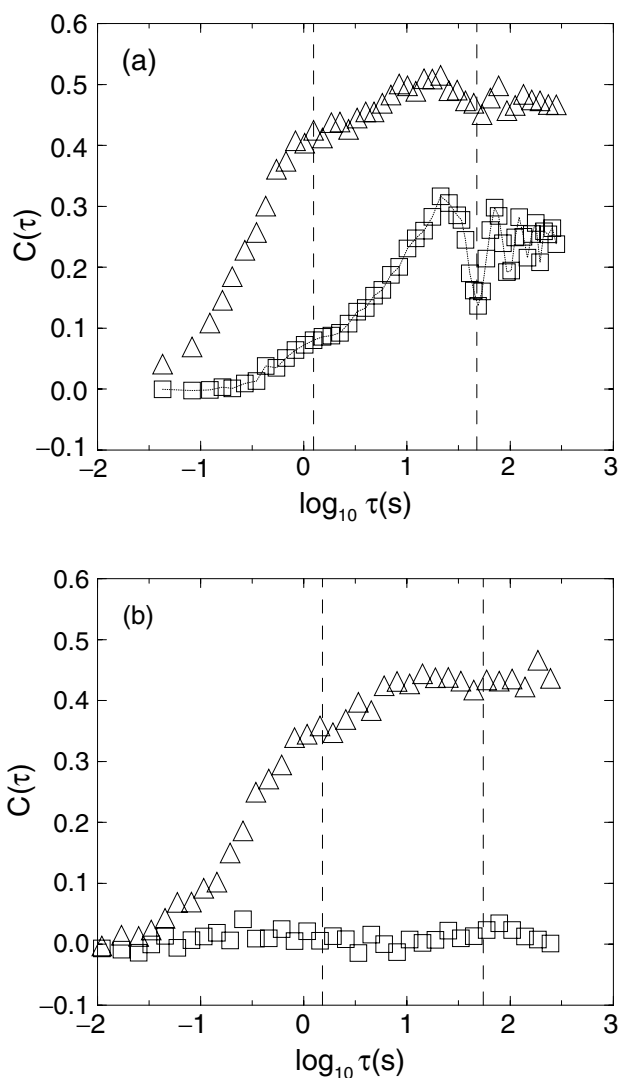
$$\tau_B = \tau_0 \frac{Nu^{1/2}}{(Ra Pr)^{1/4}} \quad (13)$$

which is easily evaluated using the measured values of  $Ra, Pr, Nu$  [26] and  $\tau_0$ . For the measurements taken at the cell centre and near the sidewall,  $Ra = 4.76 \times 10^9$ ,  $\tau_0 \approx 50$  s and  $\tau_B \approx 1.2$  s. For the measurements taken near the bottom plate,  $Ra = 3.56 \times 10^9$ ,  $\tau_0 \approx 55$  s and  $\tau_B \approx 1.5$  s.

In figure 1, we show  $C(\tau)$  as a function of  $\tau$  for the measurements taken at the cell centre. The two horizontal velocity differences are weakly correlated with the temperature difference, with  $C(\tau) \leq 0.1$  throughout the whole range of  $\tau$ . On the other hand, for the vertical velocity,  $C(\tau)$  increases with  $\tau$  when  $\tau$  is small and saturates at a value of about 0.3 when  $\tau$  becomes larger. These results thus support that the vertical velocity is distinctive due to buoyancy, reinforcing the recent finding that the vertical velocity components have statistical characteristics different from those of the horizontal velocity components [27]. Thus, buoyancy is important even at the centre of the cell where the mean temperature gradient vanishes. Moreover, we find that  $C(\tau)$  for the vertical velocity component becomes significant when  $\tau \geq \tau_B$ , further indicating that buoyancy is indeed relevant only in the Bolgiano regime.

Similar results are also found for the measurements taken near the sidewall and near the bottom plate as shown in figure 2. The correlation coefficient  $C(\tau)$  for the vertical velocity is larger than that for the horizontal velocity. Moreover, the correlation between the vertical velocity and temperature differences again increases with  $\tau$  for small  $\tau$  and saturates at a time scale, which is about 5 or 6 times larger than that of  $\tau_B$ . This supports that the buoyancy-relevant scale might vary with the location inside the convection cell [16, 28].

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**Figure 2.** Correlation coefficient  $C(\tau)$  of the temperature difference with the  $y$  component ( $\square$ - $\square$ ) and  $z$  component ( $\triangle$ - $\triangle$ ) of the velocity difference measured (a) near the sidewall and (b) near the bottom plate. The two broken lines show  $\tau_B$  and  $\tau_0$ .

The temperature fluctuations near the sidewall have been found to have a well-defined oscillation with the same period as that of the velocity oscillation [29]. This common oscillation for both the temperature and the  $y$ -component of the velocity near the sidewall might explain the relatively large value of  $C(\tau)$  for  $\tau$  close to  $\tau_0$ . Indeed, it is seen that  $C(\tau)$  oscillates around  $\tau_0$ . The physical origin of the oscillation in the  $y$ -component of the velocity is, however, not fully understood. The correlation coefficient for the vertical velocity component saturates to a value close to 0.5 near the sidewall and close to 0.4 near the bottom plate. Both values are larger than that at the cell centre. As  $\delta T$  and thus buoyancy effect is larger near the bottom plate than at the cell centre, a larger value near the bottom plate is expected. A larger value near the sidewall is also in accord with the experimental observation that heat is transported mainly by the thermal plumes moving near the sidewall than through the central region [21, 30].

### 3. Velocity and temperature cross-scaling

In this section, we show that the BO scaling implies a cross-scaling between the normalized vertical velocity and temperature structure functions. The major assumption behind the BO scaling is that in the Bolgiano regime, the temperature variance cascades and that the statistical properties of the vertical velocity difference  $w_r \equiv w(\vec{x} + \vec{r}, t) - w(\vec{x}, t)$  and the temperature difference  $T_r$  are determined by the average temperature dissipation rate  $\chi$ , the separation  $r$  and buoyancy  $\alpha g$ . Using dimensional analysis, one gets

$$w_r \sim (\alpha g)^{2/5} \chi^{1/5} r^{3/5}, \quad (14)$$

$$T_r \sim (\alpha g)^{-1/5} \chi^{2/5} r^{1/5}. \quad (15)$$

As a result,  $\tilde{W}_p(r) \equiv \langle |w_r|^p \rangle \sim r^{3p/5}$  and  $\tilde{R}_p(r) \sim r^{p/5}$ . The vertical velocity and temperature power spectra, which are related to the Fourier transforms of  $\tilde{W}_2(r)$  and  $\tilde{R}_2(r)$ , therefore have scalings in the wavenumber  $k$  as  $k^{-11/5}$  and  $k^{-7/5}$ , respectively. Besides the scaling behaviour for the vertical velocity and temperature structure functions, equations (14) and (15) also lead to various relations among the vertical velocity and temperature structure functions and their correlations [31, 32]. To find the length scales for which buoyancy is important, one requires the power injected into the flow due to buoyancy to be larger than the average energy dissipation rate:

$$\alpha g T_r w_r > \epsilon \quad (16)$$

and obtains  $r > l_B$  showing that  $l_B$  is the buoyancy-relevant scale as discussed in section 1. If one uses the locally averaged temperature dissipation rate in equations (14) and (15) (see below) and the locally averaged energy dissipation rate in equation (16), then one obtains the local version of the Bolgiano scale as the buoyancy-relevant scale.

Variations in the temperature dissipation rate are neglected in equations (14) and (15). One way to take such variations into account is to follow ideas of Kolmogorov's refined similarity hypothesis [33] and replace the globally averaged  $\chi$  by  $\chi_r$ , the local temperature dissipation rate averaged over a ball of radius  $r$ . The finding [34] that the conditional statistics of the temperature difference at fixed values of  $\chi_r$  become Gaussian and thus non-intermittent in the Bolgiano regime supports such refined similarity ideas. Eliminating  $\chi$  (or  $\chi_r$  when  $\chi$  is replaced by  $\chi_r$ ) from equations (14) and (15) leads to

$$\tilde{W}_{2p}(r) \sim (\alpha g r)^p \tilde{R}_p(r) \quad (17)$$

for any  $p$ . Thus, equation (17) remains valid even when there are intermittency corrections due to variations in  $\chi$ . It further implies

$$\frac{\tilde{W}_{2p}(r)}{[\tilde{W}_2(r)]^p} \sim \frac{\tilde{R}_p(r)}{[\tilde{R}_1(r)]^p} \quad (18)$$

or its temporal counterpart:

$$\frac{W_{2p}(\tau)}{[W_2(\tau)]^p} \sim \frac{R_p(\tau)}{[R_1(\tau)]^p} \quad (19)$$

for any  $p$ . Here  $W_p(\tau) \equiv \langle |w_\tau|^p \rangle$  where  $w_\tau \equiv w(\vec{x}, t + \tau) - w(\vec{x}, t)$ .

It was found that [17] at the cell centre, the temperature structure functions  $R_p(\tau)$  do not have discernible scaling with  $\tau$  but exhibit the so-called generalized extended self-similarity [35], a generalized scaling of the form:

$$\frac{R_p(\tau)}{[R_2(\tau)]^{p/2}} \sim \left( \frac{R_1(\tau)}{[R_2(\tau)]^{1/2}} \right)^{\rho_T(p)} \quad (20)$$

for any  $p$ . In the present study, we find that the temperature measurements taken near the sidewall and near the bottom plate of the convection cell have the same feature. Hence, equation (19) together with equation (20) imply a cross-scaling between the normalized vertical velocity and temperature structure functions:

$$\frac{W_p(\tau)}{[W_2(\tau)]^{p/2}} \sim \left( \frac{R_q(\tau)}{[R_1(\tau)]^q} \right)^{\rho_{VT}(p,q)} \quad (21)$$

for any  $p$  and  $q \neq 1$ . The cross-scaling exponents  $\rho_{VT}(p, q)$  are given by

$$\rho_{VT}^{BO}(p, q) = \frac{\rho_T(p/2) - p/2}{\rho_T(q) - q} \quad (22)$$

for any  $p$  and  $q \neq 1$ . The superscript BO emphasizes that  $\rho_{VT}^{BO}(p, q)$  in equation (22) are the values implied by the BO scaling. In particular,  $\rho_{VT}^{BO}(2p, p) = 1$  for  $p \neq 1$  as obtained directly from equation (19).

#### 4. Examination of the validity of the velocity and temperature cross-scaling

We now examine the validity of equation (19) using the measurements taken at the centre, near the sidewall and the bottom plate of the convection cell. When calculating  $W_p(\tau)$ , we correct the sampling bias of the LDV data [36] by weighing  $w_\tau$  with the minimum of the two transit times of  $w(\vec{x}, t)$  and  $w(\vec{x}, t + \tau)$ . To check whether there exists a velocity and temperature cross-scaling, we plot  $\log_{10}(W_{2p}/W_2^p)$  versus  $\log_{10}(R_p/R_1^p)$ . Typical results are presented in figure 3.

It can be seen that the data points obtained at the cell centre in the Bolgiano regime can be well described by a straight line. On the other hand, the data points obtained near the sidewall and near the bottom plate are more scattered and do not fall well on a straight line. Thus, our analyses show the existence of a velocity and temperature cross-scaling in the Bolgiano regime at the cell centre. We emphasize that this is not a trivial finding as both the vertical velocity and the temperature structure functions do not have discernible scaling in  $\tau$  at the cell centre. As clearly shown in figure 3, the data points at all the three locations cannot be described by a straight line of slope 1, showing that equation (19) is not supported by the experimental measurements.

In the following, we focus on the measurements taken at the cell centre and study the cross-scaling exponents  $\rho_{VT}(2p, p)$ . As shown in figure 4,  $\rho_{VT}(2p, p)$  is found to be a function of  $p$ , denoted by  $\gamma(p)$ . That is, we have

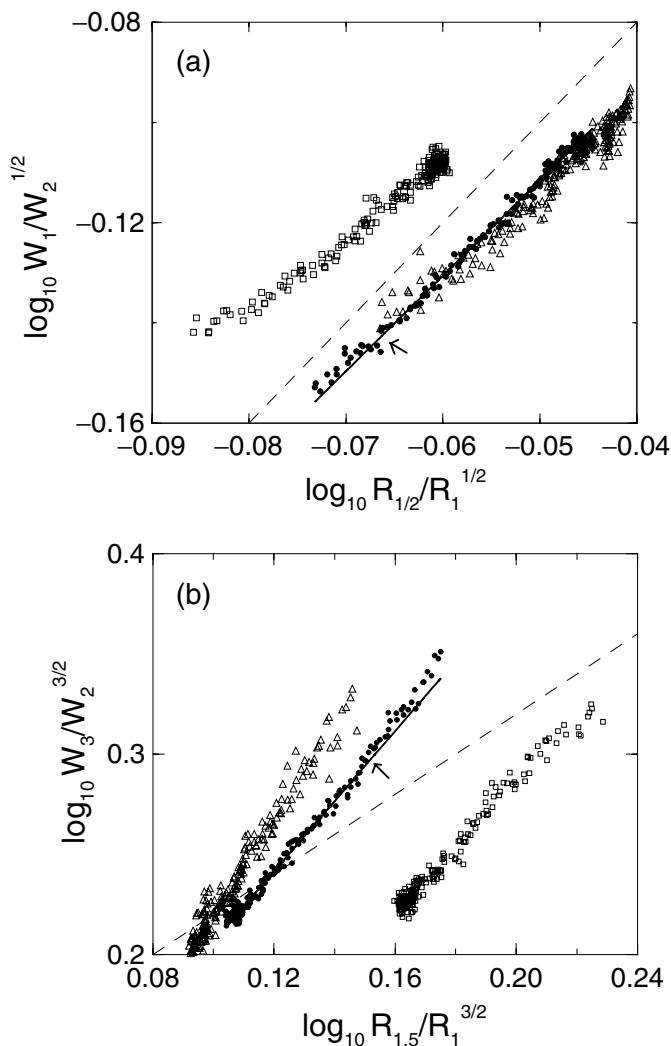
$$\frac{W_{2p}(\tau)}{[W_2(\tau)]^p} \sim \left( \frac{R_p(\tau)}{[R_1(\tau)]^p} \right)^{\gamma(p)} \quad (23)$$

with  $\gamma(p) \neq 1$  instead of equation (19). As a result, the cross-scaling exponents  $\rho_{VT}(p, q)$  should be given by

$$\rho_{VT}(p, q) = \gamma(p) \left[ \frac{\rho_T(p/2) - p/2}{\rho_T(q) - q} \right] \quad (24)$$



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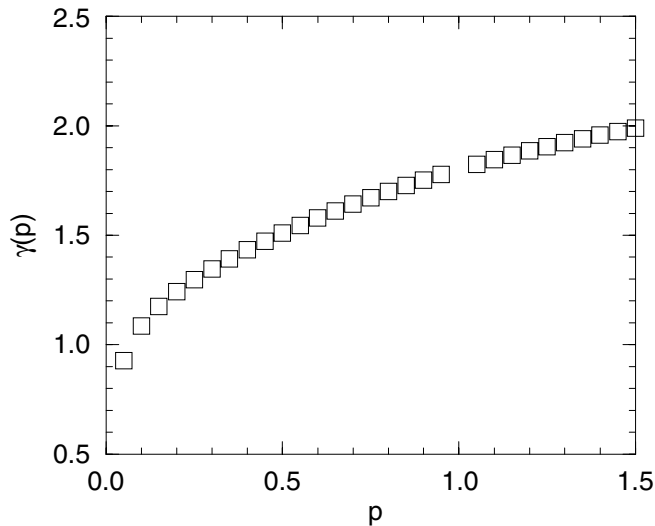


**Figure 3.**  $\log_{10}(W_{2p}/W_2^p)$  versus  $\log_{10}(R_p/R_1^{p/2})$  at the cell centre ( $\bullet$ ) for (a)  $p = 0.5$  and (b)  $p = 1.5$ . The solid line is a least-squares fit to the data points in the Bolgiano regime and is plotted for the whole range of  $\tau$ . A possible deviation of the data points from the solid line might occur around  $\tau_B$  (indicated by the arrow). The results for the measurements taken near the sidewall ( $\Delta$ ) and the bottom plate ( $\square$ ) are also shown. The dashed line is a line of slope 1.

instead of equation (22). Again, the deviation of  $\gamma(p)$  from one shows explicitly that the relation between the vertical velocity and temperature structure functions, equation (19), as implied by the BO scaling is not supported by the measurements taken at the centre of the convection cell. Our results thus cast doubt on the validity of the BO scaling in turbulent thermal convection.

### 5. Summary

We have analysed simultaneous velocity and temperature measurements taken at three representative locations: the centre, near the sidewall and near the bottom plate of the convection cell. First, we have demonstrated that the vertical velocity difference has a positive and larger



**Figure 4.**  $\gamma(p)$  as a function of  $p$  for the measurements at the centre of the convection cell.

correlation with the temperature difference in the Bolgiano regime, supporting the idea that buoyancy is important in the Bolgiano regime. The effect of buoyancy is found to be stronger near the bottom plate and near the sidewall of the cell than at the cell centre. Nonetheless, buoyancy is relevant even at the cell centre where there is a vanishing mean temperature gradient. Secondly, we have shown the existence of a cross-scaling between the normalized vertical velocity and temperature structure functions in the Bolgiano regime at the cell centre. This is a non-trivial finding as both the vertical velocity and the temperature structure functions themselves do not show discernible scaling at the cell centre. This velocity and temperature cross-scaling is implied by the BO scaling. However, we have found that the cross-scaling exponents are different from the values implied by the BO scaling. Near the sidewall and the bottom plate of the convection cell, the data points are more scattered and the existence of such a velocity and temperature cross-scaling is less certain. Nonetheless, it is clear that the data points obtained at these two locations also cannot be described by a straight line of slope 1 as implied by the BO scaling (see figure 3). Our results thus show that the validity of the BO scaling in turbulent thermal convection is far from a settled issue and more analysis is needed.

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