Journal of Turbulence
Vol. 11, No. 35, 2010, 1–10

Statistics of the locally averaged thermal dissipation rate in turbulent Rayleigh–Bénard convection

Xiaozhou He[a], Penger Tong[a] and Emily S. C. Ching[b*]

[a] *Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong;* [b] *Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Shatin, Hong Kong*

(*Received 21 December 2009; final version received 29 June 2010)*)

From the measured thermal dissipation rate in turbulent Rayleigh–Bénard convection in a cylindrical cell, we construct a locally averaged thermal dissipation rate $\chi_{f\tau}$ by averaging over a time interval τ. We study how the statistical moments $\langle(\chi_{f\tau})^p\rangle$ depend on τ at various locations along the vertical axis of the convection cell. We find that $\langle(\chi_{f\tau})^p\rangle$ exhibits good scaling in τ, of about a decade long, with scaling exponents $\mu(p)$ for $p = 1$–6. For Rayleigh number (Ra) around 8×10^9, the scaling range is 1.4–21 s at the cell center and 4–21 s at the bottom plate. The dissipative and turnover times are about 0.8 s and 35 s respectively, while the timescale corresponding to the local Bolgiano scale is estimated to be about 31 s at the cell center and 3.5 s at the bottom plate. On the basis of several assumptions, we derive theoretical predictions for $\mu(p)$ at the different locations. The measured values of $\mu(p)$ are presented and shown to be in good agreement with our theoretical predictions.

Keywords: turbulent convection

1. Introduction

Fluid turbulence is often thought of as a cascade process that transfers kinetic energy from large to small scales and in which the energy transfer rate plays a special role. Kolmogorov's 1941 theory [1] assumed that the energy transfer rate is constant and equal to the mean energy dissipation rate $\langle\epsilon\rangle$ and predicted universal statistics of $\delta v(l)$, the velocity increments between two points separated by a distance l, when l is within the inertial range. Velocity increments $\delta v(l)$ are difficult to measure, and in most experimental studies, velocity data are taken as a function of time at a fixed spatial location; as a result, the velocity temporal increments $\delta v(\tau)$ between measurements taken at a time interval τ apart, are studied instead. In turbulent flows where there is a relatively large mean flow velocity U, the correspondence between the spatial and temporal increments can be made [2,3] using Taylor's hypothesis [4] with $l = U\tau$. In such situations, it has been known [2] that the measured scaling behavior of $\delta v(\tau)$ shows a deviation from the prediction of Kolmogorov's 1941 theory (with the correspondence of l to $U\tau$ made), and a long-standing challenge in turbulence research is to understand from first principles the origin of this deviation, which is known as anomalous scaling. The observed anomalous scaling of $\delta v(\tau)$ is equivalent to

*Corresponding author. Email: ching@phy.cuhk.edu.hk

ISSN: 1468-5248 online only

DOI: 10.1080/14685248.2010.507767
http://www.informaworld.com

a τ dependence of the standardized probability density functions of $\delta v(\tau)$. Kolmogorov's refined similarity hypothesis (RSH) [5] proposed the replacement of $\langle\epsilon\rangle$ by the local energy dissipation rate ϵ_l, averaged over a region of linear scale l, and attributed the origin of the anomalous scaling of $\delta v(l)$ to the scale dependence of the statistics of ϵ_l. The temporal counterpart of Kolmogorov's RSH would then attribute the origin of anomalous scaling of $\delta v(\tau)$ to the τ dependence of the statistics of ϵ_τ, the energy dissipation rate that is averaged over a time interval τ.

Similar anomalous scaling behavior has been found in turbulent thermal convection, in that the standardized probability density functions of the temperature temporal increments $\delta T(\tau)$ [6] and velocity temporal increments $\delta v(\tau)$ [7] display a dependence on τ. In analogy to the kinetic energy cascade, thermal convective turbulence has been proposed [8] as a cascade process in which the variance of temperature fluctuations is being transferred from large to small scales at a constant rate equal to the mean thermal dissipation rate $\langle\chi\rangle$. An extension of the temporal counterpart of Kolmogorov's RSH to turbulent thermal convection leads to the proposal that the anomalous scaling in turbulent thermal convection is due to the τ dependence of the statistics of the locally averaged thermal dissipation rate over a time interval τ [9,10].

To gain insights into the cascade processes in turbulence and as a first step to investigate the validity of these refined similarity ideas, it would thus be interesting to study the statistics and particularly the scale dependence of the locally averaged energy or thermal dissipation rates directly in experiments. Directly measuring ϵ involves simultaneous measurements of the nine components of the velocity gradient tensor and is thus nontrivial. In comparison, measuring χ, which involves simultaneous measurements of the three components of the temperature gradient vector, is more manageable. Even so, direct measurements of χ were only taken recently.

In this paper, we report our study of the statistics of the locally averaged thermal dissipation rate constructed from direct measurements of χ taken in turbulent Rayleigh–Bénard convection. In Section 2, we describe the experiment and discuss our method of analysis. In particular, we explain how we construct the locally averaged thermal dissipation rate $\chi_{f\tau}$, which focuses on the contribution solely due to the fluctuation of the temperature gradient and averaged over a time interval τ, from the direct measurements of χ. We study how the statistical moments of $\chi_{f\tau}$ depend on τ and find good scaling behavior of $\langle(\chi_{f\tau})^p\rangle$ in τ for $p = 1, 2, \ldots, 6$ at various locations along the vertical axis of the cylindrical convection cell. In Section 3, we discuss how we generalize earlier work [11,10], which is based on several assumptions, to obtain theoretical predictions for the scaling exponents of the statistical moments of $\chi_{f\tau}$ at different heights from the bottom plate of the convection cell. In Section 4, we present the measured values of the scaling exponents and show that they are in good agreement with the theoretical predictions. Finally, we end this paper with a summary in Section 5.

2. Experiment and method of analysis

Turbulent Rayleigh–Bénard convection has been a system of much research interest (see, e.g., [12,13,14] for a review). In Rayleigh–Bénard convection, a closed cell of fluid is heated from below and cooled on top. For a given fluid at a fixed mean temperature in a given cell, the state of flow is characterized by the dimensionless Rayleigh number (Ra), which is defined by $\alpha g(\Delta T)H^3/(\nu\kappa)$ and measures how much the fluid is driven by the temperature difference ΔT across the height H of the cell. Here, g is the acceleration due to gravity, and α, ν, and κ are respectively the volume expansion coefficient, kinematic viscosity, and thermal diffusivity of the fluid. Previous experimental studies of turbulent Rayleigh–Bénard convection focused mainly on temperature T and velocity measurements [14]. Direct measurements of the thermal dissipation field

$$\chi(\mathbf{r}, t) = \kappa|\nabla T(\mathbf{r}, t)|^2 \tag{1}$$

were carried out only recently [15] as a function of time t and over varying Ra and spatial positions $\mathbf{r}$ across the convection cell.

The experiment was conducted in an upright cylindrical cell, of inner diameter $D =$ 19.0 cm and height $H = 20.5$ cm, filled with water. The Prandtl number (Pr = ν/κ) was kept fixed at Pr ≈ 5.5. The measurements of $\chi(\mathbf{r}, t)$ were made using a small homemade temperature gradient probe consisting of four identical thermistors, with one placed at the center of the probe and the other three placed at a short distance $\delta l = 0.25$ mm from the central one, each along the three perpendicular x, y, and z directions. We note that the shortest length scale of interest in the problem would be the thermal boundary layer thickness δ, which is known to scale as Ra, approximately as $\delta \sim \text{Ra}^{-2/7}$ [16]. Using the measurements [17] taken for Ra from 2×10^8 to 2×10^{10}, we find that along the central vertical axis, $\delta = 1.0$ mm at Ra = 1.75×10^9. Thus for the range of Ra studied, which is between 9×10^8 and 9×10^9, the separation δl is smaller than the thermal boundary layer thickness δ. All the thermistors were calibrated individually with an accuracy of about 5 mK for the temperature difference in each direction. Other details about the convection cell and the measurements of $\chi(\mathbf{r}, t)$ can be found in [15].

The temperature gradient ∇T is a sum of the mean temperature gradient and the fluctuation in the temperature gradient, which are the gradient of the mean temperature T_m and temperature fluctuation T_f respectively:

$$\nabla T(\mathbf{r}, t) = \nabla T_m(\mathbf{r}) + \nabla T_f(\mathbf{r}, t). \tag{2}$$

From its definition in Equation (1), χ contains three terms:

$$\chi(\mathbf{r}, t) = \kappa|\nabla T_m(\mathbf{r})|^2 + \kappa|\nabla T_f(\mathbf{r}, t)|^2 + 2\kappa\nabla T_m(\mathbf{r}) \cdot \nabla T_f(\mathbf{r}, t)\,. \tag{3}$$

We construct the locally averaged thermal dissipation rate by averaging over a time interval τ. We are interested in the scale dependence of the statistics of the locally averaged thermal dissipation rate, that is, how the statistical moments of the locally averaged thermal dissipation rate depend on τ. As $\nabla T_m(\mathbf{r})$ is time-independent, its average over a time interval τ is independent of τ and is thus not of interest. Therefore, to study the leading-order τ dependence of the statistics, we focus only on χ_f:

$$\chi_f(\mathbf{r}, t) \equiv \kappa|\nabla T_f(\mathbf{r}, t)|^2, \tag{4}$$

which is the contribution to χ that has the strongest dependence on time. We then construct the locally averaged thermal dissipation rate $\chi_{f\tau}$ by averaging χ_f over a time interval τ:

$$\chi_{f\tau}(\mathbf{r}, t) = \frac{1}{\tau}\int_t^{t+\tau} \chi_f(\mathbf{r}, t')\, dt'. \tag{5}$$

Then we evaluate the statistical moments $\langle(\chi_{f\tau})^p\rangle$, averaged over time, at different locations $\mathbf{r}$ and study how they depend on τ. In this study, we focus on measurements taken along the vertical axis at different heights from the bottom plate. As we shall see in Section 4, $\langle(\chi_{f\tau})^p\rangle$ exhibits good scaling in τ with scaling exponent $\mu(p)$, defined by

$$\langle(\chi_{f\tau})^p\rangle \sim \tau^{\mu(p)}\,, \tag{6}$$

for $p = 1, 2, \ldots, 6$.

3. Theoretical predictions

In this section, we obtain theoretical results for $\mu(p)$ at different vertical heights from the bottom plate of the convection cell on the basis of earlier work [11,10]. In [10], the focus is on understanding the statistics in the central region of the convection cell, and the locally averaged thermal dissipation rate was defined as the average of χ over a time interval τ and denoted as χ_τ. In the central region, $\nabla T_m \approx 0$, and thus $\chi_\tau \approx \chi_{f\tau}$. In the present work, we are also interested in the statistics at locations other than the central bulk region. In these other regions, particularly within the thermal boundary layers, ∇T_m would be dominant. As discussed in Section 1, the average of ∇T_m over a time interval τ would be independent of τ and is thus not of interest. Hence, we exclude the contributions from ∇T_m and first generalize the earlier work by refining the definition of the locally averaged thermal dissipation rate to $\chi_{f\tau}$.

The starting point of the theory is an assumption that the moments $\langle(\chi_{f\tau})^p\rangle$ have a hierarchical structure of the She–Leveque form [18]:

$$\frac{\langle(\chi_{f\tau})^{p+2}\rangle}{\langle(\chi_{f\tau})^{p+1}\rangle} = \left[\frac{\langle(\chi_{f\tau})^{p+1}\rangle}{\langle(\chi_{f\tau})^{p}\rangle}\right]^{\beta}\left[\lim_{q\to\infty}\frac{\langle(\chi_{f\tau})^{q+1}\rangle}{\langle(\chi_{f\tau})^{q}\rangle}\right]^{1-\beta}, \tag{7}$$

where β is some parameter satisfying $0 < \beta < 1$. This assumption has been checked in [11]. As direct measurements of χ were not available at that time, the temperature gradient in χ was estimated by a temperature time derivative, and the corresponding hierarchical structure was found to be valid for p around 1 for temperature measurements taken at the cell center in an experiment using low-temperature helium gas [19]. Higher-order moments could not be studied because of insufficient statistics.

This hierarchical structure [Equation (7)] implies [18] that the scaling exponents $\mu(p)$ are given by the following mathematical form:

$$\mu(p) = c(1-\beta^p) - \lambda p, \tag{8}$$

where c is the codimension of the sets of the maximum thermal dissipation rate or the so-called most dissipative structures and λ is the negative of the scaling exponent of $\langle(\chi_{f\tau})^{m+1}\rangle/\langle(\chi_{f\tau})^{m}\rangle$ as $m \to \infty$, that is,

$$\lim_{m\to\infty}\langle(\chi_{f\tau})^{m+1}\rangle/\langle(\chi_{f\tau})^{m}\rangle \sim \tau^{-\lambda}. \tag{9}$$

For $p = 1$, $\langle\chi_{f\tau}\rangle \approx \langle\chi_f\rangle = \langle\chi(\mathbf{r}, t)\rangle - \kappa\langle|\nabla T_m(\mathbf{r})|^2\rangle$, and is τ-independent. Thus $\mu(1)$ is equal to 0, and β, c, and λ are related:

$$c(1-\beta) - \lambda = 0. \tag{10}$$

The ratio of $\langle(\chi_{f\tau})^{q+1}\rangle/\langle(\chi_{f\tau})^{q}\rangle$ is dominated by the largest thermal dissipation rate as $q \to \infty$ [18] and was thus estimated [10,11] as $(\Delta T)^2/t_{r(\tau)}$. Here $t_r(\tau)$ is the timescale at the scale $r = U_0\tau$ and U_0 is some typical velocity, which is estimated by the mean velocity in regions where there is a mean flow, as near the top and bottom boundaries, and by the root-mean-squared velocity fluctuations in regions where there is no mean flow, as in the central bulk region. Take [11] $t_{r(\tau)} = r/u_r$, where u_r is the velocity fluctuation at scale r, and let $u_r \sim r^b$; then λ is related to the scaling exponent b of u_r by $\lambda = 1 - b$.

It is known that the value of b can be different, depending on whether buoyancy is significant or not. When buoyancy forces are sufficiently strong, they would directly affect the velocity statistics, and in this sense, temperature is an active scalar. In this case, u_r is expected to be related to the buoyancy term, which gives $u_r \sim r^{3/5}$ [11], that is, $b = 3/5$. On the other hand, when buoyancy is not significant, one expects $u_r \sim r^{1/3}$, that is, $b = 1/3$. Buoyancy forces are the strongest at the top and bottom plates, where the temperature gradient is the largest. As one moves away from the bottom plate toward the cell center, the effect of buoyancy becomes increasingly weaker. The second assumption of the theory is that temperature is active at the top and bottom plates and becomes passive at the cell center for moderate Ra. This assumption is consistent with the conclusion [14] that the best chance to observe Bolgiano–Obukov scaling, the scaling behavior that is believed to hold when buoyancy is significant, is close to the top and bottom plates and that Bolgiano–Obukov scaling cannot be expected at the cell center. As a result, $\lambda(z)$ is a function of the vertical distance z from the bottom plate. It increases from

$$\lambda(z = 0) = \lambda_{\text{active}} = \frac{2}{5} \tag{11}$$

at the bottom plate to

$$\lambda(z = H/2) = \lambda_{\text{passive}} = \frac{2}{3} \tag{12}$$

at the cell center and decreases back to λ_{active} as one goes beyond the central region to the top plate. When Boussinesq approximation holds, the top–bottom symmetry implies that

$$\lambda(z) = \lambda(H - z). \tag{13}$$

The sets of largest thermal dissipation rate should be along the top and bottom plates, where the temperature gradients are the strongest, and are thus quasi-two-dimensional. Hence we take the most dissipative structures to be sheetlike, and $c = 1$, near the top and bottom plates. It is unclear how the dimension of the most dissipative structures might change as one moves away from the bottom plate. Our third assumption is that they remain sheetlike such that c remains 1 as one moves away from the bottom plate along the vertical axis.

Putting these results together, we have

$$\mu(p) = 1 - [1 - \lambda(z)]^p - \lambda(z)p \tag{14}$$

along the vertical axis. Hence we predict the p dependence of $\mu(p)$ to be different at different heights from the bottom plate, and this difference along the vertical axis is solely due to the change in the strength of the buoyant forces. We shall check these theoretical predictions in the next section.

4. Results and discussions

We construct $\chi_{f\tau}$ from χ taken at different locations along the vertical axis. For the measurements taken at the center of the bottom plate, one thermistor was in contact with the plate, and the other three were at a distance δl above the plate. At each location, there are 1×10^6 to 4×10^6 data points, resulting from over 8-h long measurement at a sampling rate of 40 Hz. We evaluate $\langle(\chi_{f\tau})^p\rangle$ and find good scaling behavior of $\langle(\chi_{f\tau})^p\rangle$ with τ. The values of the scaling exponents $\mu(p)$ do not change with Ra in the Ra range studied.

To display the scaling behavior more clearly, we show the compensated plots of $\tau^{-\mu(p)}\langle(\chi_{f\tau})^p\rangle/\langle\chi\rangle^p$ versus τ for several values of p in the cell center and at the bottom plate in Figures 1 and 2 respectively. Here we normalize $\langle(\chi_{f\tau})^p\rangle$ by $\langle\chi\rangle^p$, where $\langle\chi\rangle = \langle\chi(\mathbf{r}, t)\rangle$ depends on $\mathbf{r}$.

At $\mathrm{Ra} = 8.3 \times 10^9$, the scaling range is about a decade long and ranges from 1.4 to 21 s at the cell center and from 4 to 21 s at the bottom plate. It is expected that buoyancy would be significant above a certain length scale. This length scale was first defined [20] using the mean energy and thermal dissipation rates that are averaged over the whole cell, $L_B = (\alpha g)^{-3/2}\langle\epsilon\rangle^{5/4}\langle\chi\rangle^{-3/4}$, and is known as the Bolgiano scale. However, since the energy and thermal dissipation rates, averaged over the cross section of the cell and denoted as $\epsilon(z)$ and $\chi(z)$, are different at different heights z, one should consider a local Bolgiano scale [21]: $L_B(z) = (\alpha g)^{-3/2}[\epsilon(z)]^{5/4}[\chi(z)]^{-3/4}$. In a numerical simulation at moderate Ra [22], it has been found that $L_B(0)/H \approx 0.1$ at the bottom plate and $L_B(H/2)/H \approx 0.88$ at the cell center. The turnover time τ_0 can be estimated [7] as the period of the velocity oscillations observed in the system [23], and $\tau_0 \approx 35$ s at $\mathrm{Ra} = 8 \times 10^9$. Associating τ_0 with H, we define the local Bolgiano time $\tau_B(z)$ as $\tau_0 L_B(z)/H$. Thus $\tau_B(0) \approx 3.5$ s at the bottom plate and $\tau_B(H/2) \approx 31$ s at the cell center. Moreover, the Kolmogorov viscous length $\eta \equiv (\nu^3/\langle\epsilon\rangle)^{1/4}$ is related [24] to the Nusselt number (Nu) and Ra by an exact relation (see, e.g., [12]): $\eta/H = \mathrm{Pr}^{1/2}/[\mathrm{Ra}(\mathrm{Nu} - 1)]^{1/4}$. Using the measured Nu–Ra

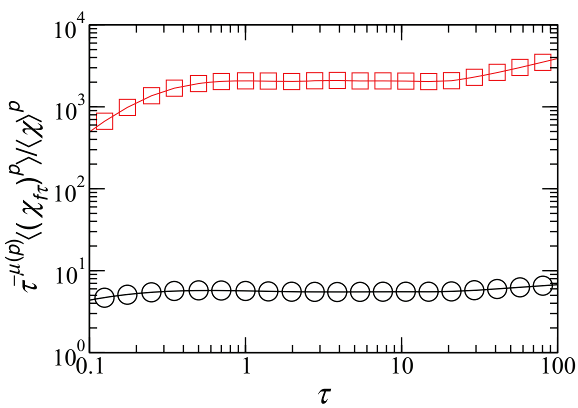

Figure 1. Plot of $\tau^{-\mu(p)}\langle(\chi_{f\tau})^p\rangle/\langle\chi\rangle^p$ as a function of τ at the cell center for $p = 2$ (circles) and $p = 4$ (squares). The measurements were taken at $\mathrm{Ra} = 8.3 \times 10^9$, and τ is in units of seconds.

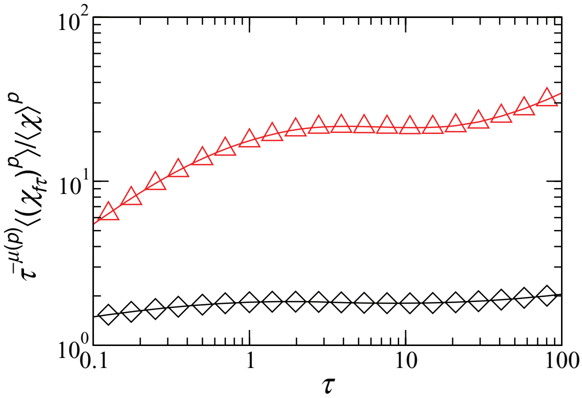

Figure 2. Plot of $\tau^{-\mu(p)}\langle(\chi_{f\tau})^p\rangle/\langle\chi\rangle^p$ as a function of τ at the center of the bottom plate for $p = 2$ (diamonds) and $p = 4$ (triangles). The measurements were taken at Ra $= 8.3 \times 10^9$, and τ is in units of seconds.

relation Nu $= 0.17$Ra$^{0.29}$ in a similar experiment [25], we obtain $\eta/H \approx 2.3 \times 10^{-3}$ at Ra $= 8 \times 10^9$. We take the dissipative length to be 10η and estimate the dissipative time τ_d, the time scale corresponding to the dissipative length, to be about 0.8 s. Thus at the cell center, the scaling range starts from around τ_d and ends before $\tau_B(H/2)$ while at the bottom plate, the scaling range starts from around $\tau_B(0)$ and ends before τ_0.

The measured values of $\mu(p)$ at different locations along the vertical axis are shown in Figure 3. We compare the values of $\mu(p)$ measured at the bottom plate with Equation (14) using Equation (11) (dashed line in Figure 3). Good agreement can be seen, confirming that $\lambda = 2/5$, the value for an active temperature, at the bottom plate. Similarly, we compare the values of $\mu(p)$ measured at the cell center with Equation (14) using Equation (12) (solid line in Figure 3), and again good agreement is found, confirming that $\lambda = 2/3$, the value

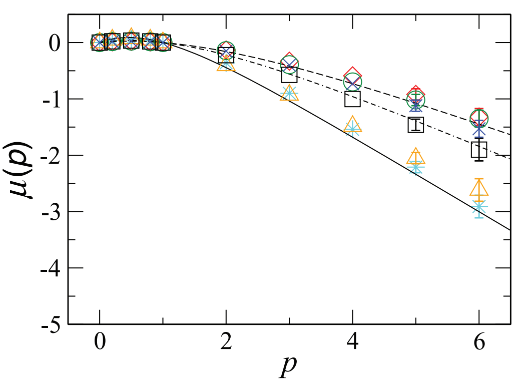

Figure 3. Values of $\mu(p)$ for different locations along the vertical axis at a distance $z = 0$ (circles), $z = 0.5\delta$ (diamonds), $z = 0.8\delta$ (pluses), $z = 1.0\delta$ (squares), and $z = 2.0\delta$ (triangles) from the bottom plate and at the cell center (stars) together with the error bars for the two largest values of p. Here Ra $= 1.75 \times 10^9$ and $\delta = 1.0$ mm is the thermal boundary layer thickness. The solid and dashed curves are the theoretical results [Equation (14)] with λ_{passive} and λ_{active} respectively, while the dot-dashed line is a fit using Equation (14) with the fitted value of $\lambda(z = 1.0\delta) = 0.47$.

for a passive temperature, at the cell center. Our results, therefore, indicate that temperature is active near the bottom plate and passive at the cell center, and this is consistent with the observation in a numerical study [22] that the local Bolgiano length, the length scale above which buoyancy is important, is smallest near the top and bottom plates and largest and comparable to the height of the cell at the cell center. It was reported in [22] that the scaling exponents of the velocity and temperature structure functions change with the height, reflecting a change in the strength of buoyancy as a function of height. Our work thus shows the interesting finding that the strength of buoyancy can also be reflected from the directly measurable scaling exponents of the moments of the locally averaged thermal dissipation rate.

As one moves away from the bottom plate toward the cell center, the p dependence of $\mu(p)$ varies, and we check that the variation is well fitted by Equation (14) with a changing $\lambda(z)$ that decreases with increasing z. Furthermore, we see that the transition of the temperature from being active to being passive is rather sharp: the measured $\mu(p)$ immediately approaches that for $\lambda = \lambda_{\text{passive}}$ once the vertical distance z from the bottom plate is beyond twice the thermal boundary layer thickness δ.

The good agreement of the measured $\mu(p)$ with our theoretical predictions thus supports the several assumptions of our theory: (1) there exists a hierarchical structure; (2) temperature is active at the bottom plate and becomes passive as one moves toward the cell center such that the value of λ is different at different heights; and (iii) the most dissipative structures remain sheetlike as one moves away from the bottom plate toward the cell center along the vertical axis [26].[1]

5. Summary

We have studied the scale dependence of the statistics of the local thermal dissipation rate $\chi_{f\tau}$, averaged over a time interval τ, using direct measurements of the thermal dissipation rate χ. In our construction of the locally averaged thermal dissipation rate, we exclude contributions from the mean temperature gradient and focus on the part that is solely due to the fluctuation of the temperature gradient. This allows us to focus on the leading-order τ or scale dependence of the statistics. We have found that the statistical moments $\langle(\chi_{f\tau})^p\rangle$ have a power-law dependence on τ with about a decade-long scaling range that varies slightly with location.

By generalizing earlier work [11,10], we have obtained theoretical predictions for the scaling exponents $\mu(p)$ at different vertical height above the bottom plate of the convection cell. There are three assumptions in our derivation. First, we have assumed that the statistical moments obey a hierarchical structure of the She–Leveque form [18]. This gives $\mu(p)$ in terms of two independent parameters c and λ [see Equation (8)]. The parameter c is the codimension of the sets of the largest thermal dissipation rates and λ is related to the scaling exponent of the velocity fluctuation at scale r with r, which, in turn, depends on whether temperature is active or passive. Our second assumption is that temperature is active at the top and bottom plates and becomes passive as one moves toward the cell center at moderate Ra. The sets of largest thermal dissipation rates are quasi-two-dimensional near the top and bottom plates. Our third assumption is that such most dissipative structures remain sheetlike as one moves away from the boundaries toward the cell center along the vertical axis. With these assumptions, we have arrived at our predictions: Equation (14) with Equations (11)–(13) for locations along the vertical axis of the convection cell. We note that the variation in the p dependence of $\mu(p)$ along the vertical axis is predicted to be the sole result of the weakening of the buoyancy effects as one moves away from the boundaries. The measured values of $\mu(p)$ confirm our theoretical predictions (see Figure 3). The measurements further show that for Ra about 10^9, the transition of temperature from being active to passive is rather sharp, occurring over a short distance of about twice the thermal boundary layer thickness.

Our present study has concentrated on positions along the vertical axis of the convection cell. For other regions of the cell, we expect λ to depend on the distance from the bottom plate only, as the strength of the buoyancy is the same at the same height. On the other hand, c might be different when one moves away from the vertical axis toward the sidewall region. Finally, we comment that whenever the extension of the temporal counterpart of Kolmogorov's RSH to turbulent convection is valid, $\mu(p)$ would be directly related to the scaling exponents of the temporal velocity and temperature structure functions. These issues will be addressed in future studies.

Acknowledgments

The authors acknowledge support by the Hong Kong Research Grants Council under grant nos. CA05/06.SC01 (ESCC and PT), CUHK-400708 (ESCC), and HKUST-602907 (PT).

Note

1. We note that the statistics of a passive scalar in a wind tunnel have also been studied using the hierarchical structure model, and the corresponding parameter for the codimension of the most intermittent structures was reported to be 0.8 ± 0.1 (see [26]).

References

[1] A.N. Kolmogorov, *The local structure of turbulence in incompressible viscous fluid for very large Reynolds number*, Dokl. Akad. Nauk. SSSR, 30 (1941), pp. 9–13. Reproduced in Proc. R. Soc. Lond., Ser. A 434 (1991), pp. 9–13.

[2] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov*, Cambridge University Press, Cambridge, 1995.

[3] K.R. Sreenivasan and R.A. Antonia, *The phenomenology of small-scale turbulence*, Annu. Rev. Fluid Mech. 29 (1997), pp. 435–472.

[4] G.I. Taylor, *The spectrum of tarbulence*, Proc. R. Soc. Lond., Ser. A 164 (1938), pp. 476–490.

[5] A.N. Kolmogorov, *A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number*, J. Fluid Mech. 13 (1962), pp. 82–85.

[6] E.S.C. Ching, *Probability for temperature differences in Rayleigh-Bénard convection*, Phys. Rev. A 44 (1991), pp. 3622–3629.

[7] E.S.C. Ching, C.K. Leung, X.-L. Qiu, and P. Tong, *Intermittency of velocity fluctuations in turbulent thermal convection*, Phys. Rev. E 68, (2003), pp. 026307-1–6.

[8] V.S. L'vov, *Spectra of velocity and temperature fluctuations with constant entropy flux of fully developed free-convective turbulence*, Phys. Rev. Lett. 67 (1991), pp. 687–690.

[9] E.S.C. Ching and K.L. Chau, *Conditional statistics of temperature fluctuations in turbulent convection*, Phys. Rev. E 63 (2001), pp. 047303-1–4.

[10] E.S.C. Ching, *Scaling laws in the central region of confined turbulent thermal convection*, Phys. Rev. E 75 (2007), pp. 056302-1–6.

[11] E.S.C. Ching and C.Y. Kwok, *Statistics of local temperature dissipation in high Rayleigh number convection*, Phys. Rev. E 62 (2000), pp. R7587–7590.

[12] E.D. Siggia, *High Rayleigh number convection*, Annu. Rev. Fluid Mech. 26 (1994), pp. 137–168.

[13] G. Ahlers, S. Grossmann, and D. Lohse, *Heat transfer and large scale dynamics in turbulent Rayleigh-Bénard convection*, Rev. Mod. Phys. 81 (2009), pp. 503–537.

[14] D. Lohse and K.-Q. Xia, *Small-scale properties of turbulent Rayleigh-Bénard convection*, Annu. Rev. Fluid Mech. 42 (2010), pp. 335–364.

[15] X.-Z. He and P. Tong, *Measurements of the thermal dissipation field in turbulent Rayleigh-Bénard convection*, Phys. Rev. E 79 (2009), pp. 026306-1–4.

[16] A. Belmonte, A. Tilgner, and A. Libchaber, *Boundary layer length scales in thermal turbulence*, Phys. Rev. Lett. 70 (1993), pp. 4067–4070.
[17] S.-L. Lui and K.-Q. Xia, *Spatial structure of the thermal boundary layer in turbulent convection*, Phys. Rev. E 57 (1998), pp. 5494–5503.
[18] Z.-S. She and E. Leveque, *Universal scaling laws in fully developed turbulence*, Phys. Rev. Lett. 72 (1994), pp. 336–349.
[19] M. Sano, X.-Z. Wu, and A. Libchaber, *Turbulence in helium-gas free convection*, Phys. Rev. A 40 (1989), pp. 6421–6430.
[20] A.S. Monin and A.M. Yaglom, *Statistical Fluid Mechanics*, MIT Press, Cambridge, MA, 1975.
[21] R. Benzi, F. Toschi, and R. Tripiccione, *On the heat transfer in Rayleigh-Bénard systems*, J. Stat. Phys. 93 (1998), pp. 901–918.
[22] E. Calzavarini, F. Toschi, and R. Tripiccione, *Evidences of Bolgiano-Obhukhov scaling in three-dimensional Rayleigh-Bénard convection*, Phys. Rev. E 66 (2002), pp. 016304-1–4.
[23] X.-L. Qiu and P. Tong, *Onset of coherent oscillations in turbulent Rayleigh-Bénard convection*, Phys. Rev. Lett. 87 (2001), pp. 094501-1–4.
[24] S. Cioni, S. Ciliberto, and J. Sommeria, *Temperature structure functions in turbulent convection at low Prandtl number*, Europhys. Lett. 32 (1995), pp. 413–418.
[25] Y.-B. Du and P. Tong, *Turbulent thermal convection in a cell with ordered rough boundaries*, J. Fluid Mech. 407 (2000), pp. 57–84.
[26] G. Ruiz-Chavarria, C. Baudet, and S. Ciliberto, *Scaling laws and dissipation scale of a passive scalar in fully developed turbulence*, Physica D 99 (1996), pp. 369–380.